The return on everything and the business cycle in production economies

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March 18, 2022
preliminary version
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JEL classification: C63, E32, E44, G12.
Keywords: Equity premium puzzle, housing, rare disasters, production CAPM, real business cycles

Abstract

Motivated by recent empirical evidence on the returns on equity, bonds, and housing, we study the interaction between an economy's total risky capital portfolio consisting of housing and equity, the business cycle, and different types of productivity risk: standard, long-run, and disaster risk. Preferences include habits or follow a generalized recursive form. Procyclical housing adjustments reduce consumption risk, preventing the versions with habits or long-run risk from simultaneously replicating risk premia, investment volatility, and housing demand. The disaster risk version replicates these targets. In all versions, a perfect negative correlation between equity returns and the marginal utility of consumption excludes Sharpe ratios of housing larger than for equity.

Acknowledgment: We are grateful to our academic advisor Alfred Maußner for initial ideas and ensuing discussions. The paper significantly benefits from comments during the Autumn Forum "Globalization of Real Estate Markets" at Zurich University, the 11th ReCapNet conference "Real Estate Asset Pricing" at ZEW Mannheim, the TU Munich Research Seminar in Economics, and the 29th BGPE Research Workshop in Nuremberg.

1 Introduction

Consumption capital asset pricing models (CCAPMs) of production economies have made great progress over the last decades in simultaneously explaining asset prices and business cycle statistics. The literature has developed several approaches that account for the equity premium puzzle raised by Mehra and Prescott (1985). However, most models focus on equity and ignore housing, which accounts for 50% of wealth in advanced economies. Neglecting housing is critical for the explanatory power of these models for two reasons. First, a high reward for holding risk also implies a high marginal propensity for hedging consumption risk. Yet, adjustments in the allocation of income between housing and non-durable consumption provides such a hedge. On the one hand, a high marginal propensity to reduce consumption risk may explain the puzzlingly elastic and procyclical demand for housing, evidenced by a highly volatile, procyclical co-movement of house prices and residential investment. On the other hand, this additional channel to reduce consumption risk may diminish the ability of existing approaches to generate a sizeable risk-reward. Second, multiple assets with different Sharpe ratios imply separate conditions for the Hansen-Jagannathan bound (HJB) and the joint distribution of consumption risk and risk premia. These conditions challenge existing approaches to reproduce Sharpe ratios. This study, therefore, addresses these two challenges for CCAPMs in production economies by considering equity and housing simultaneously.

Our framework is a standard real business cycle (RBC) model that features housing services as a durable consumption good. Key elements for risk premia are the risk process and the model's stochastic discount factor. Accordingly, we consider different versions of both elements that are known to successfully reproduce equity premia. We thereby concentrate on productivity and growth risk as the only source of uncertainty. Next to a standard productivity process, we allow long-run productivity risk as in Bansal and Yaron (2004) and Croce (2014) and disaster risk as in Rietz (1988), Barro (2006), and Gourio (2012). The stochastic discount factor (SDF) follows either from Chen's (2017) version of Campbell and Cochrane's (1999) external habit formation or from a generalized recursive utility introduced by Epstein and Zin (1989) and Weil (1989). In addition to the two challenges described above, our study also rechecks the compatibility of representative-agent production-based CCAPMs with the data on housing.

In response to the first object of investigation, i.e., whether these models can replicate sizeable risk premia and the procyclical demand for housing, and to recheck their general compatibility with housing, we confront the models with stylized facts extracted from data from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) (JKKST) and OECD.Stats (2019). Business cycle statistics reveal the following well-known characteristics: i) residential investments are at least moderately more volatile than business investments, ii) house prices are at least twice as volatile as Gross Domestic Product (GDP), and iii) house prices, business investments, and GDP correlate positively with residential investments, and house prices and GDP are also positively correlated. Turning to asset prices, the stylized facts are i) a stable risk-free rate smaller than 2.25 percent, ii) return rates on equity moderately larger than on housing, iii) risk premia all larger than 3 percent, iv) return rates and premia on equity which are at least twice as volatile as return rates and premia on housing and on total risk, and as a result, v) a Sharpe ratio of housing significantly larger than of equity.

In response to the second object of investigation, i.e., how a second risky asset with a different Sharpe ratio restricts possible explanations for the observed Sharpe ratio of equity, we decompose the Sharpe ratio into two factors: i) the SDF's coefficient of variation (the HJB) and ii) the correlation between the asset's risk premia and the SDF. The HJB then defines a common upper bound for the Sharpe ratios of all assets. The second factor is asset-specific and determines the relative size of Sharpe ratios. Thus, while only the size of the product of the two factors matters for the size of the Sharpe ratio of a single asset, the presence of a second asset with a different Sharpe ratio introduces separate bounds for each factor individually.

We find that a model with standard productivity risk and external habits can replicate housing demand and equity premia, but underestimates housing premia and the volatility of business investment. Further, risk premia on both assets correlate perfectly negative with the model's SDF. Consequently, the Sharpe ratios of both assets are almost equal to the HJB. The model's HJB is too small and Sharpe ratios are underestimated. Conversely, the equity premium relies on counterfactually volatile return rates. The same holds for the already too small housing premia.

A model with long-run productivity risk and Epstein and Zin (1989) preferences replicates equity premia but underestimates housing premia, the volatility of house prices and the volatility of both investment goods. Yet, compared to the previous model, premia are now generated in a different way. The HJB is larger and compensates less volatile return rates. Further, the model dissolves the nearly perfect correlation between return rates and the SDF. Unfortunately, at odds with the data, housing premia in the model correlate distinctly less with the SDF than premia on equity. Thus, the Sharpe ratio of housing falls below the Sharpe ratio of equity and, compared to the data, the Sharpe ratio of equity turns out too large while the Sharpe ratio of housing is too small.

Finally, the model with disaster risk reproduces the volatility and the co-movements of GDP, business investments, residential investments, and house prices. The model replicates the observed demand for housing together with other business cycle statistics, even though the variables partly correlate too strongly. With disaster risk, risk premia are larger than in the other versions. The model predicts an equity premium that exceeds the value observed in the U.S. by 1.7 percentage points. The housing premia is below its empirical value by the same amount. Premia in the model rely on a HJB that is similar to the model with long-run risk. The perfect negative correlation between equity premia and the SDF is dissolved to some degree, yet, the correlation still remains far too large and rules out additional assets with substantially larger Sharpe ratios. Although the correlation between housing premia and the SDF increases compared to the model with long-run risk, it still falls below the correlation between equity premia and the SDF. As a result, the Sharpe ratio of equity is larger than in the data, whereas the Sharpe ratio of housing is too small. If simulations exclude disasters, the decomposition of Sharpe ratios into the HJB and correlations between premia and the SDF does not hold on average since the simulated distribution deviates from the expected distribution. In these simulations, the Sharpe ratio of housing exceeds the Sharpe ratio of equity, although equity premia correlate stronger with the SDF than housing premia. However, the Sharpe ratio of housing still remains too small, whereas the Sharpe ratio of equity remains too large.

Regarding our first question, all models retain their ability to generate sizeable equity premia despite introducing housing as a second asset. Additionally, the version with external habits and the version with disaster risk reproduce the empirically observed demand for housing solely by productivity uncertainty. However, procyclical housing demand provides an additional hedge against consumption risk. All variants underestimate housing premia and, apart from disaster risk, sizeable equity premia can only be explained if investment variability is restricted, resulting in a too smooth business investment activity. Disasters in the model destroy a part of the capital and housing stock, so that disaster risk limits the possibilities to reduce risk through housing. Moreover, time-varying risk for a large depreciation of the housing stock introduces an additional demand effect for housing. An increase in the disaster probability increases expectations about future stock depreciation and thus leads the household to deinvest. Without changes in productivity, the decreasing demand for housing decreases house prices. This effect is sizeable and helps to increase the overall volatility and co-movement of housing-related prices and quantities.

By answering our second question, we identify the following shortcomings of the present CCAPM framework for explaining asset prices. First, equity premia and the stochastic discount factor correlate too strongly. Therefore, the Sharpe ratio of equity is close to its upper bound, and any asset with a significantly larger Sharpe ratio is impossible to explain within this setting. Second, the HJB bound is too small to facilitate the replication of the Sharpe ratio of housing.. Third, productivity risk as the only source of uncertainty results in stronger correlations between equity premia and the SDF than between housing premia and the SDF. As long as the realized distribution of the shocks meets the expected one, the Sharpe ratio of housing falls below the Sharpe ratio of equity.

There is extensive literature covering each two of the three following topics: housing, production economies inside a Dynamic Stochastic General Equilibrium (DSGE) framework, and asset prices in CCAPM. We contribute to the literature in the following way by integrating all three topics into one common framework.

First, standard RBC models that feature housing as in Davis and Heathcote (2005) are doomed to fail in replicating the observed demand for housing. This strand of literature adds productivity shocks to the construction sector to account for the high volatility of residential investments. Yet, this leads to a counterfactual negative co-movement of housing-related prices and quantities. Thus, Iacoviello and Neri (2010) conclude that productivity uncertainty alone cannot account for the demand for housing and establish housing demand shocks. Nguyen (2018) and Fehrle (2019) solve the co-movement puzzle by increasing the income effect for housing with adjustment costs for the stock of housing and capital, but neither accounts for the high house price volatility. Khan and Rouillard (2018) consider habits in consumption in combination with borrowing constraints and conclude that productivity uncertainty is insufficient to reproduce house price volatility. Dorofeenko et al. (2014) study higher-order productivity uncertainty combined with bankruptcy costs and replicate the house price volatility at the expense of underestimated residential and business investment volatility. Chahrour and Gaballo (2020) assume that households use house prices to assess the macroeconomic situation. This assumption creates a channel where rising house prices increase the demand for houses, which raises house prices again. The upward spiral then qualitatively explains the demand effect for housing.[1] Our framework offers simple approaches that can explain the demand for housing with productivity or time-varying disaster risk solely.

Second, several authors studied CCAPMs with equity and housing in endowment economies. Jordà et al. (2019) argue that several approaches that are successful in reproducing equity premia are less successful once they take housing and total wealth into account. Piazzesi et al. (2007) consider housing as an asset that enters the household's consumption basket. The authors study the interaction of asset prices and risk in the composition of the consumption basket. Fillat (2009) expands the Piazzesi et al. (2007) framework by generalized recursive utility and long-run risk. However, different from our framework of a production economy, endowment economies do not attempt to explain the behavior of quantities and exclude any possibility for hedging consumption risk.

Lastly, we find that the progress of CCAPMs in production economies sparked by Jermann (1998) relies on an excessive correlation between equity premia and the SDF. Moreover, procyclical residential investments reduce consumption risk and thereby complicate the explanation of sizeable risk premia. This mechanism is similar to the insurance opportunity against consumption risk provided by endogenous decisions about labor supply on Walrasian labor markets, as previously discussed in the literature. However, hedging consumption risk through the labor market is questionable: households are usually forced into unemployment during downturns and do not voluntarily substitute consumption with leisure in order to smooth the bundle and reduce consumption risk as in the model. If hedging through the labor market is restricted, labor market statistics and sizeable risk premia can be explained simultaneously (see Boldrin et al., 2001; Uhlig, 2007; Heer and Maußner, 2013). The question, whether housing hedges consumption risk is discussible. On the one hand, Iacoviello (2005), Lustig and van Nieuwerburgh (2005), Mian et al. (2013), and Mian and Sufi (2014) suggest quite the opposite. The authors even motivate consumption risk on imperfect capital markets through declining house prices: declining house prices increase the leverage ratio and tightened (re)financing options force the household on the margin to reduce its consumption. On the other hand, the literature provides evidence that households reallocate non-durable consumption and residential investment to keep the composite of non-durable consumption and housing smooth. For example, Piazzesi et al. (2007) and Khorunzhina (2021) argue that households prefer the intratemporal substitution between housing and non-durable consumption to the intertemporal substitution of the whole composite. In addition, Khorunzhina (2021) outlines that while homeowner's expenditures on maintenance, repairs, and improvements of houses are sizeable on average, they are incurred infrequently at the individual household level: the average annual expenditures amount to 1.6% of the house value, while their within household's coefficient of variation is 108%. The observed elastic and procyclical demand for housing would then be consistent with the assumption that households realize their infrequent but sizeable investments during good times rather than in bad times. This behavior would be similar to the piece-by-piece construction of houses in developing countries. In those countries,

[1] Chahrour and Gaballo (2020) do not undertake a full quantitative assessment of the model.

housing serves as a saving stock due to incomplete financial markets (Rosling et al., 2019, Chapter 6). We do not contribute to the debate which effect prevails, but the literature has not taken the latter sufficiently into account.

Jaccard (2011) and Favilukis et al. (2017) already examine risk premia in production economies with housing. The model of Jaccard (2011) is similar to our model with external habits and standard productivity risk. However, its empirical targets rest on Piazzesi et al. (2007), who assume that the house price index grows with the price index of residential investments, whereas Davis and Heathcote (2007) and Knoll et al. (2017) show that the main driver for increasing house prices are land prices. In consequence, Jaccard (2011) relies on empirical targets where housing returns are markedly smaller than equity returns and where the Sharpe ratios of the two assets are similar. In contrast, JKKST, Campbell et al. (2009), and Demers and Eisfeldt (2021) report similar return rates for housing and equity, but a Sharpe ratio of housing significantly higher than the one of equity. Favilukis et al. (2017) depart from the representative-agent framework and study a production economy with two sectors and aggregated as well as idiosyncratic income risk. Their model explains the boom-bust cycle in the first decade of this century and matches the empirical Sharpe ratio of equity, although the mean and the standard deviation of the return rates turn out moderately too small. Further, the model replicates a sizeable risk premium for housing, yet the authors do not report the Sharpe ratio of the housing index.

From here on, the paper reads as follows. In section 2, we present the stylized facts on which the remainder of the paper focuses and discuss the suitability of the JKKST data for our purposes. Section 3 presents the basic framework of our RBC model with housing, while section 4 closes the model with different specifications of productivity risk and the SDF. We present and discuss the results in section 5. The paper concludes with section 6. The appendix collects additional data work and more detailed derivations.

2 Stylized facts

We start with the presentation of stylized facts which characterize historical data on business cycles, housing, and asset prices and which the literature has identified as main facts that are commonly valid for most countries for extended periods (see JKKST for asset prices and Davis and Nieuwerburgh (2015) for housing and business cycles). In Table 1, we provide a summary of these stylized facts for the US (1970-2015), the UK (1969-2015), France (1980-2015), and Japan (1963-2015). Asset price statistics are annual data from the JKKST database. Business cycle statistics are quarterly data from OECD.Stats (2019).[2]

Panel A of Table 1 shows the stylized facts from the housing and the business cycle literature. We observe that GDP has a standard deviation of approximately 1.5-1.6 percent in the US, the UK, and Japan and slightly below 1 percent in France. In the US and the UK, residential investments are twice as volatile as business investments. The difference between the two volatilities is moderately smaller in Japan and significantly smaller in

[2] While later we target US data, we provide here a broader set of countries to evidence non-country specific facts. Next to these four countries, Appendix A shows that we also observe the same stylized facts in most of the 16 developed countries JKKST examine.

France.[3] In all four countries, the ratio of the standard deviation of business investment to the standard deviation of GDP lies between 2.4 and 2.9, and the ratio of the standard deviation of house prices to the standard deviation of GDP is larger than 2. GDP, house prices, residential and business investment co-move. The lowest correlation is observed between business and residential investments in the US. In short, investment quantities and house prices co-move procyclically. Usually, the literature additionally considers lagged cross-correlations with residential investments since residential investments lead the business cycle in the US. However, Kydland et al. (2016) show that this is unique to the US and Canada, wherefore we omit lead-lag patterns here.

Panel B of Table 1 displays the return rates on bills, on the two risky assets, equity and housing , on total risk, and the risk premia. Moreover, Panel B also shows the corresponding standard deviations and the resulting Sharpe ratios of equity, housing, and total risk. We observe a low ”risk-free” return rate on bills between 0.98 percent and 2.24 percent with a low standard deviation (2.3-3.7). The returns on equity are between 5.86 percent and 9.61 percent, and result in equity premia between 4.88 percent and 7.37 percent. In all countries, the average return on housing turns out moderately lower than the average return on equity, and housing premia are between 3.54 percent and 5.44 percent. The difference between the two risky returns/premia is 0.32, 1.00, 1.43, and 3.83 percentage points in Japan, the US, the UK, and France, respectively.[4] Moreover, in the US and the UK, the return on total risk is approximately the average of the two risky return rates. In France, the return on total risk is close to the smaller return on housing, and in Japan, the return on total risk exceeds the decomposed return rates on both risky assets. While returns on equity exceed returns on housing moderately, they are two to four times as volatile: the standard deviation of equity returns lies between 16.7 and 24.11, while the standard deviation of housing returns falls between 3.78 and 9.65. In all countries, the standard deviation of returns on total risk is also significantly lower than the standard deviation of returns on equity. Risk premia are almost identically volatile as return rates. In all countries, the Sharpe ratio of housing exceeds the Sharpe ratio of equity significantly, and the Sharpe ratio of total risk is close to the Sharpe ratio of housing.

There is a certain dissent in the literature about housing returns, and some authors report lower returns on housing than JKKST. For example, Eichholtz et al. (2021) report a return on housing of 4.0% in Paris during 1809 – 1943 and 4.8% in Amsterdam during 1900 –1979. Chambers et al. (2021) find a return on housing of 2.3% for the residential real estate portfolios of four large Oxbridge colleges during 1901 – 1983. By contrast, several other studies also support the results of JKKST. Demers and Eisfeldt (2021) find a nominal net return rate of 8.5% for Single-family rentals in the U.S. during the more recent period 1986 – 2014; close to the nominal net housing return rate of 8.86% in the JKKST database during the same period. Further, Demers and Eisfeldt (2021) report a Sharpe ratio of 1.14 for housing and blame the mere focus of previous studies on rental yields or house price appreciation for the lower returns previously reported. Campbell et al. (2009)

[3]For most continental European countries we observe the same relation as in France.

[4]The difference between the return rates in France is closer to the value in the other countries in the periods chosen by JKKST (1963-2015 and 1870-2015). Our French data set starts in 1980 due to missing data for the business cycle statistics.

Table 1: Empirical returns, premiums and second moments

	USA	UK	France	Japan
Panel A: Business Cycle				
$\sigma(GDP)$	1.52	1.58	0.95	1.59
$\frac{\sigma(I)}{\sigma(GDP)}$	2.91	2.68	2.75	2.41
$\frac{\sigma(D)}{\sigma(GDP)}$	6.85	5.56	3.17	3.84
$\frac{\sigma(P_H)}{\sigma(GDP)}$	2.03	4.85	3.19	2.70
$\rho(P_H, D)$	0.67	0.51	0.65	0.31
$\rho(I, D)$	0.07	0.16	0.64	0.27
$\rho(GDP, D)$	0.72	0.69	0.81	0.45
$\rho(GDP, P_H)$	0.64	0.71	0.48	0.55
Panel B: Return Rates				
R_g	1.57	1.56	2.24	0.98
R_E^{lev}	7.45	8.00	9.61	5.86
R_H	6.01	7.00	5.78	5.54
R_T^{lev}	6.84	7.47	6.61	6.19
EP	5.88	6.44	7.37	4.88
HP	4.45	5.44	3.54	4.56
TP	5.27	5.91	4.37	5.21
$\sigma(R_g)$	2.31	3.73	2.55	2.53
$\sigma(R_E^{lev})$	16.71	23.41	24.11	20.15
$\sigma(R_H)$	3.78	9.64	5.52	6.53
$\sigma(R_T^{lev})$	6.90	8.44	6.95	8.10
$\sigma(EP)$	16.47	24.27	23.98	19.94
$\sigma(HP)$	4.41	8.88	6.18	6.47
$\sigma(TP)$	7.00	8.62	7.39	8.03
SR_E	0.36	0.27	0.31	0.24
SR_H	1.01	0.61	0.57	0.70
SR_T	0.75	0.69	0.59	0.65

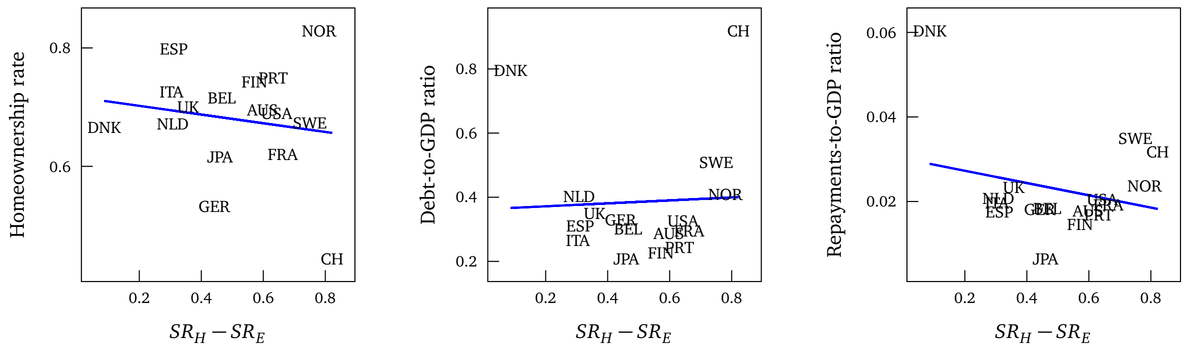
Notes: Periods: USA 1970-2015, United Kingdom 1969-2015, France 1980-2015, and Japan 1963-2015.
Business Cycle Moments: Standard deviations $\sigma(.)$ and correlations $\rho(.,.)$ for GDP, business investments I, residential investments D and house prices P_H. Business cycle statistics are computed for logged and hp-filtered (1600) quarterly per-capita data. Main source: OECD.Stats (2019), own calculations, Appendix A provides more information.
Return Rates: Mean percentage returns on equity R_E^{lev}, housing R_H, total risk R_T^{lev} and government bonds R_g, as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP), the corresponding standard deviations $\sigma(.)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). Asset price statistics are computed for annual data. Source: JKKST, own calculations, Appendix A provides more information.

find housing premia of 3% in the U.S. during the recent period 1975 – 2007, which is somewhat lower than the 4.45% reported in Table 1. Nevertheless, Campbell et al. (2009) report a housing premia standard deviation of 3.13, which results in a similar Sharpe ratio of 0.96.

The JKKST data includes national return rates aggregated from owner-occupied units. This methodology casts doubt on the validity of the reported housing Sharpe ratios for our framework for two reasons. First, it may be questionable whether homeowners are marginal housing investors over the business cycle since they already invested at the extensive margin. Second, the aggregated data lacks information on idiosyncratic risk, although idiosyncratic risk may account for a significant share of the risk of owner-occupied units—particularly when compared to easily diversifiable equity. Within the representative agent framework, data on returns of investible housing units to a diversified and deep-pocketed investor may seem more suitable than returns on owner-occupied units.

Concerning the first issue, Khorunzhina (2021) argues that the average cross-sectional and intra-household variation of homeowners' residential investments are substantial, and so are the changes in their housing stock. Hence, homeowners are intensive marginal housing investors.

Regarding the second issue, firstly, Demers and Eisfeldt (2021) report a Sharpe ratio of 1.14 for Single-family rentals. Single-family rentals add up to 35% of U.S. rental housing units and thus, are investible assets for a diversified and deep-pocketed investor. Hence, their results suggest a similar housing Sharpe ratio that may be considered more consistent to the representative-agent-framework. Secondly, if idiosyncratic risk is the main driver for different Sharpe ratios of housing and equity, there would be a high willingness to pay for hedging and diversifying. Against this backdrop, the reasons for not more predominate housing supply by diversified landlords and for capital markets failing to offer any hedge would be puzzling. For example, a contract of difference or an asset swap could hedge local price risk using a national house price index as underlying, and nationwide acting housing cooperatives could diversify local price risk. Finally, Figure 1 illustrates the relationship between the excess Sharpe ratio of housing over equity and different indicators for idiosyncratic risk, namely, the homeownership rate, the mortgage debt-to-GDP ratio, and the mortgage repayment-to-GDP ratio. The homeownership ratio serves as an indicator for the portfolio diversification of investors: e.g. if all housing units belonged to the same owner, this investor is exposed to no local price risk, and a homeownership rate of one would imply no diversification against local price risk at all. The mortgage debt-to-GDP ratio indicates the average level of leverage. Since a higher leverage ratio amplifies the effects of idiosyncratic risk, we interpret the mortgage debt-to-GDP ratio as an instrument for the impact of idiosyncratic risk. Similarly, given that the mortgage repayment-to-GDP ratio indicates debt sustainability and thus debtors' resilience to idiosyncratic shocks, we understand the ratio as an instrument for measuring the consequences of idiosyncratic shocks. If idiosyncratic risk was the only reason for the excess Sharpe ratio of housing, a lower degree of diversification, a larger impact, and more severe consequences of idiosyncratic shocks should correlate positively with the excess Sharpe. However, positive correlations are not evident, but rather negative correlations between the excess Sharpe



(a) Homeownership rate (b) Mortgage debt-to-GDP (c) Mortgage repayment-to-GDP

Main source: JKKST, own calculations, Appendix A provides more information.

Figure 1: Idiosyncratic risk and the housing Sharpe ratio excess

ratio for housing and the idiosyncratic risk indicators.[5]

Summing up, we conclude that other factors than idiosyncratic risk must be substantial for the excessive Sharpe ratio of housing. This conclusion complements Jordà et al. (2019). They outline that the sheer size of the Sharpe ratio excess and persistence over different horizons makes it unlikely that idiosyncratic risk is its only driver. One last note, the often-used return rates from real estate investment trusts are not comparable to the JKKST housing returns. These trusts often invest in commercial real estate additionally and are typically highly leveraged, whereas the JKKST returns apply to non-leveraged investments in residential real estate.

3 Basic Framework

Next, we introduce the basic framework of our analysis. We consider an economy that consists of an infinitely-lived representative household and a representative firm. Time is discrete and indexed by $t \in \mathbb{N}$.

Representative Firm The representative firm produces output Y_t from labor N_t and capital services K_t according to a Cobb-Douglas production function

$$Y_t = Z_t (A_t N_t)^{1-\alpha} K_t^{\alpha}, \quad \alpha \in (0,1). \tag{1}$$

Total factor productivity Z_t and labor augmenting technical progress A_t may be stochastic and the exact form of the stochastic processes characterizing productivity risk will be pinned down in chapter 4. However, A_t will grow at the rate $a > 0$ in the deterministic versions of the models in all scenarios.

[5] As we do not control for other factors, this does not mean that housing is free of idiosyncratic risk but, by definition, that the housing Sharpe ratio excess depends on other factors as well and not *only* on idiosyncratic risk.

The firm's first-order conditions from maximization of profits $Y_t - W_t N_t - r_{K,t} K_t$ under perfect competition and subject to the production function read

$$W_t = (1-\alpha)\frac{Y_t}{N_t}, \tag{2a}$$

$$r_{K,t} = \alpha \frac{Y_t}{K_t}. \tag{2b}$$

Representative Household The representative household derives utility from streams $\{\tilde{C}_t\}_{t\in\mathbb{N}}$ of a composite good

$$U_0 = U(\{\tilde{C}_t\}_{t\in\mathbb{N}}) \tag{3}$$

The composite good consists of consumption C_t, housing H_t and leisure $1-N_t$ and will be specified more concretely below.

The household supplies labor services N_t and capital services K_t to the firm and receives wages W_t and capital rents $r_{K,t}$. It buys consumption goods C_t and invests business investments I_t in productive capital and residential investments D_t in new homes. Hence, its budget constraint reads

$$W_t N_t + r_{K,t} K_t = C_t + I_t + D_t. \tag{4}$$

Capital evolves according to

$$K^*_{t+1} = (1-\delta_K)K_t + \Phi\left(\tfrac{I_t}{K_t}\right)K_t, \tag{5a}$$

$$K_{t+1} = e^{\omega_{t+1} b_{t+1}} K^*_{t+1}, \tag{5b}$$

where $\delta_K \in (0,1)$ is the depreciation rate. Additionally, b_t is binary and indicates disasters ($b_t = 1$) and normal times ($b_t = 0$). In line with Gourio (2012), the fraction $1-e^{\omega_t}$ of the capital stock destructs each period during disasters. Chapter 4 characterizes the stochastic properties of b_t and ω_t. However, in scenarios without disaster risk, $b_t \equiv 0$ and $e^{\omega_t b_t} \equiv 1$ holds for all t. The function $\Phi : (0,\infty) \to \mathbb{R}$ describes adjustment costs to the capital stock in the manner of Jermann (1998)

$$\Phi(x) = b_1 + \tfrac{b_2}{1-\kappa}x^{1-\kappa}, \quad b_1 \in \mathbb{R}, b_2 \in (0,\infty). \tag{6}$$

On the other hand, residential investments must be combined with a fixed factor L of land normalized to one to form new houses, where $\varphi \in (0,1)$ is the share of land

$$H_{\text{new},t} = D_t^{1-\varphi} L^{\varphi}.$$

The stock of houses then evolves according to

$$H^*_{t+1} = (1-\delta_H)H_t + H_{\text{new},t}, \tag{7a}$$
$$H_{t+1} = e^{\omega_{t+1} b_{t+1}(1-\varphi)} H^*_{t+1}, \tag{7b}$$

where $\delta_H \in (0,1)$ is the depreciation rate of houses and disasters destruct the fraction $1-e^{\omega_t}$ of residential structures.[6]

Finally, we assume that the consumption bundle $\tilde{C}_t$ is of the Cobb-Douglas form, i.e.,

$$\tilde{C}_t = C_t^{\mu_C}(A^{\varphi}_{t-1}H_t)^{\mu_H}(A_{t-1}(1-N_t))^{\mu_N}, \quad \mu_C + \mu_H + \mu_N = 1. \tag{8}$$

The fact that we multiply housing H_t, which grows at the rate $a^{1-\varphi}$, with A^{φ}_{t-1} and leisure $1-N_t$ with A_{t-1}, ensures that a balanced growth path exists even if the bundle was a more general CES aggregate. While the weighting with the level of productivity is not necessary for the special case of a Cobb-Douglas bundle, it nonetheless helps to increase risk-reward. Hence, we follow Croce (2014) with this assumption and interpret the weighting with adjustments in the standard of living.

The household chooses consumption C_t, its labor supply N_t, business investments I_t, next period's pre-destructed capital stock K^*_{t+1}, residential investments D_t, and next period's pre-destructed housing stock H^*_{t+1} to maximize its lifetime utility under the budget constraint (4) and the dynamics (5) and (7). It takes wages W_t and the rental rate $r_{K,t}$ of capital as given. Hence, the first-order conditions of the household can be summarized as

$$W_t = MRS_t^{N,C}, \tag{9a}$$
$$q_t = \mathbb{E}_t\left[e^{\omega_{t+1}b_{t+1}}M_{t,t+1}\left(r_{K,t+1} + q_{t+1}\left(1-\delta_K + \Phi\Big(\frac{I_{t+1}}{K_{t+1}}\Big) - \Phi'\Big(\frac{I_{t+1}}{K_{t+1}}\Big)\frac{I_{t+1}}{K_{t+1}}\right)\right)\right], \tag{9b}$$
$$P_{H,t} = \mathbb{E}_t\left[e^{\omega_{t+1}b_{t+1}(1-\varphi)}M_{t,t+1}\left(r_{H,t+1} + P_{H,t+1}(1-\delta_H)\right)\right], \tag{9c}$$

where $MRS_t^{N,C} = \frac{\partial \tilde{C}_t/\partial(1-N_t)}{\partial \tilde{C}_t/\partial C_t} = \frac{\mu_N}{\mu_C}\frac{C_t}{1-N_t}$ is the marginal rate of substitution between leisure and consumption, $r_{H,t} = \frac{\partial \tilde{C}_t/\partial H_t}{\partial \tilde{C}_t/\partial C_t} = \frac{\mu_H}{\mu_C}\frac{C_t}{H_t}$ is the implicit rental rate of housing derived from the marginal rate of substitution between housing and consumption, and $M_{t,t+1}$ is the model's SDF. Moreover, $q_t = \frac{1}{\Phi'(\frac{I_t}{K_t})}$ is Tobin's q and $P_{H,t} = \frac{1}{1-\varphi}D_t^{\varphi}$ is the relative price of new houses.

General Equilibrium In general equilibrium, the first-order conditions (2) and (9) of the firm and the household hold, production is determined by (1), and the stocks of capital and houses evolve according to (5) and (7). Consumption, business investments, and residential investments are homogenous goods aggregated in output Y_t. Hence, the economy's

[6] Land does not depreciate and disasters do not destruct land. See Davis and Heathcote (2005) for the link between the depreciation of residential structures and houses.

resource constraint is[7]

$$Y_t = C_t + I_t + D_t. \tag{10}$$

Finally, we follow Davis and Heathcote (2005) and define GDP as output plus the implicit rent from housing by

$$GDP_t = Y_t + r_{H,t} H_t.$$

Return Rates The return rate $R_{E,t+1}$ on equity, the return rate $R_{H,t+1}$ on housing, and the return rate $R_{T,t+1}$ on total risk are given by

$$\begin{aligned}
R_{E,t+1} &= e^{\omega_{t+1} b_{t+1}} \frac{r_{K,t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta_k + \Phi(\frac{I_{t+1}}{K_{t+1}}))}{q_t} - 1 \\
&= \frac{r_{K,t+1} K_{t+1} - I_{t+1} + q_{t+1} K^\star_{t+2}}{q_t K^\star_{t+1}} - 1, \qquad \text{(11a)} \\
R_{H,t+1} &= e^{(1-\varphi)\omega_{t+1} b_{t+1}} \frac{r_{H,t+1} + P_{H,t+1}(1 - \delta_H)}{P_{H,t}} - 1 \\
&= \frac{r_{H,t+1} H_{t+1} - P_{H,t+1} H_{new,t+1} + P_{H,t+1} H^\star_{t+2}}{P_{H,t} H^\star_{t+1}} - 1, \qquad \text{(11b)} \\
R_{T,t+1} &= \frac{r_{K,t+1} K_{t+1} - I_{t+1} + q_{t+1} K^\star_{t+2} + r_{H,t+1} H_{t+1} - P_{H,t+1} H_{new,t+1} + P_{H,t+1} H^\star_{t+2}}{q_t K^\star_{t+1} + P_{H,t} H^\star_{t+1}} - 1 \\
&= \frac{q_t K^\star_{t+1}}{q_t K^\star_{t+1} + P_{H,t} H^\star_{t+1}} R_{E,t+1} + \frac{P_{H,t} H^\star_{t+1}}{q_t K^\star_{t+1} + P_{H,t} H^\star_{t+1}} R_{H,t+1} \qquad \text{(11c)}
\end{aligned}$$

Since stock returns provide the basis for the observed return on equity, it includes leverage. This does not hold for housing returns. To be in line with the data, we also consider leveraged return rates. More precisely, we assume that in each period the constant fraction $m \in [0, 1)$ of the firm's capital stock is financed by debt through bonds which all have maturity τ. Next to these corporate (c) bonds, we also consider government (g) bonds. As in Barro (2006) and Gourio (2012) bonds may default during disasters. The bonds differ by their loss given default $1 - \Gamma_{g,t}$ and $1 - \Gamma_{c,t}$ which are specified in chapter 4. The price of a bond with maturity τ and recovery rate Γ_{t+1} then follows the recursion

$$Q^{(\tau)}_{j,t} = \mathbb{E}_t[M_{t,t+1}(1 - b_{t+1}(1 - \Gamma_{j,t+1}))Q^{(\tau-1)}_{j,t+1}], \quad \text{where } Q^{(\tau)}_{j,t+1} = 1 \text{ and } j \in \{g, c\}.$$

Further, the ex-post return rate from holding a bond with maturity τ for one period is

[7]The economy's resource constraint already implies the budget constraint (4) of the household in equilibrium since the firm makes no profits.

defined by

$$R^{(\tau)}_{j,t+1} = \frac{(1-b_{t+1}+b_{t+1}\Gamma_{\tau-1,t+1})Q^{(j)}_{j,t+1}}{Q^{(\tau)}_{j,t}} - 1.$$

Since the Modigliani and Miller theorem holds, the leveraged return rate on equity and total risk are given by

$$R^{lev}_{E,t+1} = \frac{1}{1-m} R_{E,t+1} - \frac{m}{1-m} R^{(c)}_{T,t+1} \tag{11d}$$

$$R^{lev}_{T,t+1} = \frac{q_t K^*_{t+1}}{q_t K^*_{t+1} + P_{H,t} H^*_{t+1}} R^{lev}_{E,t+1} + \frac{P_{H,t} H^*_{t+1}}{q_t K^*_{t+1} + P_{H,t} H^*_{t+1}} R_{H,t+1}. \tag{11e}$$

Lastly, when talking about the return rate $R_{g,t+1}$ on a government bond we mean the return on a bond with the maturity of one period

$$R_{g,t+1} = R^{(1)}_{g,t+1}. \tag{11f}$$

Fundamental Requirements Using the insights from Lucas (1978) and Hansen and Jagannathan (1991), the Sharpe ratios of equity and housing satisfy

$$SR_E := \frac{\mathbb{E}\left[R^{lev}_{E,t+1} - R_{g,t}\right]}{\sigma\left[R^{lev}_{E,t+1} - R_{g,t}\right]} = -\text{CV}[M_{t,t+1}]\,\text{Corr}\left[M_{t,t+1}, R^{lev}_{E,t+1} - R_{g,t}\right] \tag{12a}$$

and

$$SR_H := \frac{\mathbb{E}\left[R_{H,t+1} - R_{g,t}\right]}{\sigma\left[R_{H,t+1} - R_{g,t}\right]} = -\text{CV}[M_{t,t+1}]\,\text{Corr}\left[M_{t,t+1}, R_{H,t+1} - R_{g,t}\right] \tag{12b}$$

where $\text{CV}[M_{t,t+1}]$ is the coefficient of variation of the model's SDF

$$\text{CV}[M_{t,t+1}] =: \frac{\sigma\left[M_{t,t+1}\right]}{\mathbb{E}\left[M_{t,t+1}\right]}, \tag{12c}$$

commonly known as the HJB. With rational expectations this bound defines a common upper bound for Sharpe ratios of all assets in the models, while different correlations between the SDF and risk premia of the assets are necessary to explain different Sharpe ratios.

More precisely, we can formulate the following quantitative requirements to replicate U.S. data. First, the HJB in the model must be at least as large as the empirical Sharpe ratio of housing (1.01). Second, the correlation between premia on housing and the SDF must be (in absolute value) approximately 3 times as large as the correlation between premia on equity and the SDF to replicate the difference in the size of those Sharpe ratios.

It follows that the correlation between premia on equity and the SDF cannot exceed 1/3.

4 Stochastic Discount Factor and Productivity Risk

In the previous section, we have specified the model's framework apart from the stochastic discount factor $M_{t,t+1}$, the processes Z_t and A_t driving productivity risk, and the process for the variables b_t, ω_t, and $1-\Gamma_{j,t}$ determining disaster risk. However, risk premia in the model heavily depend on these features. We, therefore, examine different versions of these elements to close the model.

4.1 Stochastic Discount Factor

We consider two specifications for the model's SDF by assuming either habits as in Chen (2017) or a recursive utility as introduced by Epstein and Zin (1989) and Weil (1989).

4.1.1 Superficial External Habits

In this version of the model, we additionally assume that leisure does not enter the consumption bundle, i.e., $\mu_N = 0$ in (8).[8] Consequently, equations (2a) and (9a) from the general framework are replaced by

$$N_t = 1.$$

The household's preferences are additive time separable and per period preferences are described by CRRA utility. The household forms habits $\tilde{C}_{h,t}$ in the consumption bundle. Thus, the utility in (3) becomes

$$U_0 = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{(\tilde{C}_t - \tilde{C}_{h,t})^{1-\gamma}-1}{1-\gamma}\right].$$

We follow Chen (2017) and assume that the surplus from the consumption bundle over the habit evolves according to

$$\ln\left(\frac{\tilde{C}_{t+1}-\tilde{C}_{h,t+1}}{\tilde{C}_{t+1}}\right) = (1-\rho_{\tilde{C}})\ln(\bar{S}_{\tilde{C}}) + \rho_{\tilde{C}}\ln\left(\frac{\tilde{C}_t-\tilde{C}_{h,t}}{\tilde{C}_t}\right) + \left(\frac{1}{\bar{S}_{\tilde{C}}}-1\right)\left(\ln\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)-a\right),$$

where a is the growth rate of the bundle on the balanced growth path of the model[9] and $\bar{S}_{\tilde{C}}$ is the steady state surplus of the consumption bundle. The corresponding stochastic

[8] On the one hand, different labor market assumptions are problematic for comparison across models. On the other hand, it is also problematic to change labor market assumptions compared to the original form of the model when studying the pure effect of adding housing. As the paper motivates the latter, we assume exogenous labor decisions here as in the original version of Chen (2017).

[9] This is ensured by the weighting of housing with A_{t-1}^{φ} in the bundle (8).

discount

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\mu_C - 1} \left(\frac{H_{t+1}}{H_t} \right)^{\mu_H} \left(\frac{\tilde{C}_{t+1} - \tilde{C}_{h,t+1}}{\tilde{C}_t - \tilde{C}_{h,t}} \right)^{-\gamma}.$$

4.1.2 Recursive Utility

Here, we assume that the household's preferences over streams of the composite good are described by a recursive utility function, as introduced by Epstein and Zin (1989) and Weil (1989), of the form

$$\tilde{V}_t = \left[(1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t \tilde{V}_{t+1}^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where ψ is the household's elasticity of intertemporal substitution (EIS) and γ the coefficient of relative risk aversion (RRA). Note, however, that γ and ψ describe the household's RRA and EIS with respect to the composite good $\tilde{C}$. Since the composite good is of the Cobb-Douglas type, the consumption-based RRA is given by $\mu_c \gamma$ and the consumption-based EIS reads $\frac{1}{1-\mu_c(1-1/\psi)}$.[10] For easier notation we define $V_t := \tilde{V}_t^{1-1/\psi}$ which satisfies the recursion

$$V_t = (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}},$$

where we use, similar to Caldara et al. (2012), the notation

$$\theta := 1 - \frac{1-\gamma}{1-\frac{1}{\psi}}.$$

In the case where $\theta = 0$, the RRA equals the reciprocal of the EIS and the household's utility reduces to the 'classical' expected discounted sum of within period CRRA utilities. Hence, θ can also be interpreted as the deviation from this 'classic' case.

With these assumptions, the model's SDF is

$$M_{t,t+1} = \beta \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{1-\frac{1}{\psi}} \frac{C_t}{C_{t+1}} \left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}} \right)^{-\theta}.$$

4.2 Productivity Risk

We introduce productivity risk into the model in three different ways by specifying the form of total productivity Z_t and labor augmenting technical progress A_t in the production function (1). Next to a standard process for productivity, we consider the effects of long-run productivity risk as in Croce (2014) and of disaster risk similar to Gourio (2012).

[10] See Swanson (2012) and Heiberger and Ruf (2019).

4.2.1 Standard Productivity Risk

For the version with standard productivity risk, log total factor productivity follows a standard AR(1) process

$$\ln Z_{t+1} = \rho_z \ln Z_t + \epsilon_{z,t+1}, \quad \epsilon_{z,t+1} \sim iid\mathcal{N}(0, \sigma_z^2).$$

Further, labor augmenting technical progress grows deterministically at the rate $a > 0$, i.e.,

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = a.$$

Lastly, there is no disaster risk, i.e. $b_t \equiv 0$ for all t so that ω_t and $\Gamma_{j,t}$ are dropped from the model.

4.2.2 Long-Run Productivity Risk

In this version, total factor productivity is non-stochastic and normalized to unity

$$Z_t = 1,$$

Labor augmenting technical progress A_t grows stochastically as in Croce (2014) according to the process

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = a + x_t + \epsilon_{a,t+1}, \tag{13a}$$

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t+1}, \tag{13b}$$

where

$$\begin{pmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{pmatrix} \sim iid\mathcal{N}(0, \Sigma), \quad \text{and } \Sigma = \begin{pmatrix} \sigma_a^2 & \rho_{a,x}\sigma_a\sigma_x \\ \rho_{a,x}\sigma_a\sigma_x & \sigma_x^2 \end{pmatrix}.$$

Shocks $\epsilon_{a,t+1}$ affect the growth rate only once in the period of occurrence and describe short-run growth risk. On the other hand, x_t describes persistent changes in the growth rate and is therefore interpreted as a long-run risk component of productivity.

As there is no disaster risk, $b_t \equiv 0$ for all t and ω_t and $\Gamma_{j,t}$ are again dropped from the model.

4.2.3 Disaster Risk

As already mentioned, disasters are introduced through the binary variable b_t which indicates disasters in the case of $b_t = 1$ while $b_t = 0$ in normal times. Next to the destruction of the fraction $1 - e^{\omega_{t+1}}$ of residential structures and capital in (5b) and (7b), disasters also result in a decline of productivity by the factor $1 - e^{\omega_{t+1}}$. Hence, technology grows stochastically according to

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = a + x_{t+1} + \omega_{t+1} b_{t+1}, \tag{14a}$$

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t+1}, \quad \epsilon_{x,t} \sim iid\mathcal{N}(0, \sigma_x^2). \tag{14b}$$

There are two differences compared to the previously introduced case of long-run risk in (13). First, the effect of the "long-run" component x on the growth rate is no longer lagged by one period but appears immediately. For our calibration with $\rho_x = 0$ used below for disaster risk, $\epsilon_{x,t+1}$, therefore, acts just as the short-run risk $\epsilon_{a,t+1}$ in (13). Second, instead of the normally distributed short-run risk $\epsilon_{a,t+1}$ the process now includes the possible effect ω_{t+1} on the growth rate in case a disaster $b_{t+1} = 1$ occurs.

Following Gourio (2012), disasters appear with time-varying probability and size. More specifically, we assume that

$$P(b_{t+1} = 1 | b_t = 0) = \min\{p_t, 1\} \quad \text{and} \quad P(b_{t+1} = 0 | b_t = 0) = 1 - \min\{p_t, 1\}$$

where the log of p_t follows an AR(1)-process

$$\ln p_{t+1} = (1 - \rho_p) \ln \bar{p} + \rho_p \ln p_t + \epsilon_{p,t+1}, \; \epsilon_{p,t} \sim iid\mathcal{N}(0, \sigma_p^2).$$

Moreover, disasters remain persistent with probability no less than $q \in (0, 1)$ so that

$$P(b_{t+1} = 1 | b_t = 1) = \max\{q, \min\{p_t, 1\}\}, \; P(b_{t+1} = 0 | b_t = 1) = 1 - \max\{q, \min\{p_t, 1\}\}.$$

Finally, the disaster size $1 - e^{\omega_{t+1}}$ also evolves stochastically according to

$$\omega_t := \bar{\omega} e^{\hat{\omega}_t},$$

$$\hat{\omega}_{t+1} = \rho_\omega \hat{\omega}_t + \epsilon_{\omega,t+1}, \quad \epsilon_{\omega,t} \sim iid\mathcal{N}\left(0, \sigma_\omega^2\right),$$

where $\bar{\omega} < 0$. We slightly deviate from the treatment in Gourio (2012) in the specification of the process governing the disaster size and allow autocorrelation but restrict outcomes to $\omega_t < 0$ so that disasters always have negative effects. This specification is similar to Fernández-Villaverde and Levintal (2018).[11] It is assumed that the shocks ϵ_x, ϵ_p, and ϵ_ω are stochastically independent.

Further, we assume that the loss given default during disasters is coupled to the disaster size $1 - e^{\omega_{t+1}}$ via constant fractions $\chi_g, \chi_c \in [0, 1]$ so that

$$1 - \Gamma_{g,t+1} = \chi_g(1 - e^{\omega_{t+1}}) \quad \text{and} \quad 1 - \Gamma_{c,t+1} = \chi_c(1 - e^{\omega_{t+1}}).$$

[11] Gourio (2012) additionally considers a transitory component of disasters. We checked the effects of a transitory shock component as well. Since we find that the effects for our targets are marginal, we omit the transitory component for the sake of simplicity.

5 Results

We proceed to analyze the models' ability to replicate the stylized facts from the data discussed in section 2. We report quantitative results for the following three model variations that combine the specifications of the SDF and of the productivity risk from section 4: external habits with standard productivity risk and recursive utility with long-run productivity risk or disaster risk.[12]

5.1 Calibration

We begin with the numeric calibration of the joint framework and add the parameters that are specific to the three versions afterwards.

5.1.1 Joint Framework

We identify one period in the model with one quarter in the data and summarize the calibration of the joint framework in panel A of Table 2.

We assume an average quarterly growth rate a of 0.5% as in Jermann (1998) and Gourio (2012) and close to the value of 0.45% used by Croce (2014) and Chen (2017). We take the share of capital $\alpha = 0.34$ in the production function from Gourio (2012) and Croce (2014), which is again almost identical to the value of 0.35 in Chen (2017). The depreciation rates of capital $\delta_K = 0.022$ and housing $\delta_H = 0.009$ are taken from Nguyen (2018), who strips down the Davis and Heathcote (2005) model. The share of land in housing matches the upper bound of $\varphi = 0.3$ from Fehrle (2019). We follow Jermann (1998) and set the parameters b_1 and b_2 in the adjustment costs function in such a way that they do not affect the model's balanced growth path. The weight of leisure μ_N in the consumption bundle is determined so that, on average, the household works one-third of its time; except for the model with external habits, where leisure does not enter the consumption bundle ($\mu_N = 0$). The weight of housing μ_H, in turn, is determined so that, on average, 19% of the households' total expenditures for consumption and housing are spent on housing (see Grossmann et al., 2021). Finally, we set the level of leverage for equity to $m = 0.3$ and the maturity of corporate bonds to 10 years.

We do not pin down the values of the discount factor β and of the elasticity κ of Tobin's q. Instead, we choose the values for each version separately in such way that we minimize the sum of squared deviations between the data and the model implied values for the following targets: the return rate of government bonds, the equity premium, the housing premium, the Sharpe ratios of the equity and housing premia and all business cycle moments from Table's 1 Panel A. We allow values in $[0; 7]$ for κ and values in $[\frac{0.99}{a_u}; \frac{0.9999}{a_u}]$ for β, where a_u is the growth factor of utility in the specific model variation.

[12] We consider long-run productivity risk and disaster risk only in combination with recursive Epstein and Zin (1989) preferences. The additional term in the SDF involving future lifetime utility for these preferences is more suitable to capture long term risk effects (see also Bansal and Yaron (2004), Kaltenbrunner and Lochstoer (2010), Rudebusch and Swanson (2012) and Gourio (2012)).

Table 2: Calibration

Parameter	Value or Target	Description
Panel A: Common Framework		
a	$\ln(1.005)$	growth rate
α	0.34	capital share in production
δ_K	0.022	depreciation rate of capital
δ_H	0.009	depreciation rate of housing
φ	0.30	share of land in housing
b_1	$\Phi(\frac{I}{K}) = a - 1 + \delta_K$	adjustment cost parameter
b_2	$\Phi'(\frac{I}{K}) = 1$	adjustment cost parameter
μ_C	$1 - \mu_H - \mu_N$	weight of consumption in bundle
μ_H	$\frac{r_H H}{C + r_H H} = 0.19$	weight of housing in bundle
μ_N	$N = 0.33$ or $\mu_N = 0$	weight of leisure in bundle
m	0.30	leverage level of equity
T	40	maturity of corporate bonds
Panel B: External Habits and Standard Productivity Risk		
γ	2	coefficient of relative risk aversion
$\bar{S}_{\tilde{C}}$	0.07	steady state surplus over habit
$\rho_{\tilde{C}}$	0.98	habit persistence
ρ_z	0.98	TFP persistence
σ_z	0.012	standard deviation of TFP innovations
		Optimized Values
β	0.9981	discount factor
κ	4.5172	elasticity of Tobin's q
Panel C: EZ Utility and Long-Run Risk		
γ	10	coefficient of relative risk aversion
ψ	2	elasticity of intertemporal substitution
ρ_x	$0.8^{\frac{1}{4}}$	persistence of long-run component of productivity
$\rho_{a,x}$	0	correlation of shocks to short- and long-run component of productivity
σ_a	$\frac{0.0335}{2}$	conditional standard deviation of long-run component of productivity
σ_x	$0.1\sigma_a$	conditional standard deviation of short-run component of productivity
		Optimized Values
β	0.9935	discount factor
κ	3.9018	elasticity of Tobin's q
Panel D: EZ Utility and Disaster Risk		
γ	3.8	coefficient of relative risk aversion
ψ	2	elasticity of intertemporal substitution
ρ_x	0.00	autocorrelation of log technology shock
ρ_ω	0.00	autocorrelation of log disaster size
ρ_p	0.90	autocorrelation of log disaster probability
σ_x	0.01	conditional standard deviation of log technology shock
$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	2.80	unconditional standard deviation of log disaster probability
$\bar{p}\exp(\frac{\sigma_p^2}{2(1-\rho_p^2)})$	0.0072	mean disaster probability
q	0.91	probability for disaster persistence
χ_g	0.20	default loss of government bonds as fraction of disaster size
χ_c	0.38	default loss of corporate bonds as fraction of disaster size
		Optimized Values
β	0.9941	discount factor
κ	1.79	elasticity of Tobin's q
$\bar{\omega}$	-0.044	disaster size
σ_ω	0.261	conditional standard deviation of log disaster size

Notes: We optimize the elasticity κ of Tobin's q over the range $[0; 7]$. The discount factor β is optimized over the range $[\frac{0.99}{a_u}; \frac{0.9999}{a_u}]$, where a_u is the growth factor of utility in the model.
With disaster risk, we additionally optimize $\bar{\omega}$ over $[-0.055; -0.025]$ and σ_ω over $[0.05; 0.55]$.

Moreover, we allow for different degrees of risk aversion (and EIS) across the three versions of the model. While this limits comparability for the different model versions, the original papers show that different degrees of risk aversion were necessary to explain equity premia. We do not want to limit the models' performance with regard to risk premia by changing the degree of risk aversion but study the effects of introducing housing when sticking to the original degree of risk aversion instead.

5.1.2 Specific Parameters

External Habits and Standard Productivity Risk The risk aversion, the habit formation, and the process driving total factor productivity are chosen identical to Chen (2017) and collectively summarized in Panel B of Table 2. The optimized values of the two free parameters are $\beta = 0.998$ and $\kappa = 3.52$.

Epstein-Zin Preferences and Long-Run Productivity Risk The calibration of the additional parameters for Epstein and Zin (1989) preferences and long-run productivity risk is summarized in Panel C of Table 2 and closely follows Croce (2014).

More concretely, the coefficient of relative risk aversion and the elasticity of intertemporal substitution are chosen as in Croce (2014), i.e., we set $\gamma = 10$ and $\psi = 2$. Further, we adjust the calibration in Croce (2014) for the stochastic process governing productivity to quarterly values. Finally, the values for β and κ, which provide the best fit of the model to data, turn out $\beta = 0.9935$ and $\kappa = 3.90$.

Epstein-Zin Preferences and Disaster Risk Lastly, Panel D of Table 2 shows the calibration of the parameters that are specific to the model version with disaster risk, which now closely follows Gourio (2012).

The elasticity of intertemporal substitution $\psi = 2$ remains at the same value as before as it already matches the value used in Gourio (2012). We adjust the coefficient of relative risk aversion to the value of $\gamma = 3.8$ from Gourio (2012) for the model with disaster risk.

We set $\rho_x = 0$ and $\sigma_x = 0.01$ so that during normal times the stochastic process governing technological progress is identical to the process for the permanent component of productivity in Gourio (2012). We choose the same autocorrelation ($\rho_p = 0.90$) and standard deviation ($\sigma_p = 2.8\sqrt{1-\rho_p^2}$) as Gourio (2012) for the disaster probabilities. Further, we set $\bar{p}$ such a way that the average probability of entering a disaster is 0.72 percent, and pin down the persistence of disasters to $q = 0.91$—the same values used by Gourio (2012). In accordance with Gourio (2012) we assume an iid process for the disaster size, i.e., $\rho_\omega = 0$. Yet, while Gourio (2012) considers a permanent and a transitory effect of disasters on productivity, our model specification only includes a permanent effect. We therefore deviate from Gourio (2012) and add the mean disaster size $\bar{\omega}$ and the standard deviation σ_ω to the list of parameters over which we optimize the model's fit to the data. The resulting values are $\beta = 0.9941$, $\kappa = 1,79$, $\bar{\omega} = -0.044$ and $\sigma_\omega = 0.261$. For comparison, Gourio (2012) assumes a mean of −0.007 and −0.055 for the effects of disasters on the permanent and transitory components of productivity, respectively.

5.2 Quantitative Model Predictions

Table 4 summarizes the quantitative results for the three versions. Panel A of the table reports the business cycle statistics, while Panel B shows the return rates generated by the model. All moments reported are the mean outcome from 100 model simulations, where each simulation includes 180 periods after 1000 burn-in periods. Business cycle moments are from logged HP-filtered (1600) simulated time series. The model solutions we use for simulations are from projection methods (see the Appendix for details).

External Habits The second column of Table 4 presents the results for the model with external habits. The model matches closely the volatility of GDP (1.41 in the model vs. 1.52 in the data), residential investments (7.44 in the model vs. 6.85 in the data), and house prices (2.33 in the model vs. 2.03 in the data). However, the relative standard deviation of business investments in the model (1.01) is too low by a factor of almost three compared to the data (2.91).

Turning to the return rates, the model replicates the return on equity fairly well (8.14% in the model and 7.45% in the data). Yet, the return on government bonds in the model (3.04%) exceeds the empirical value (1.57%) by more than one percentage point. Consequently, the model can explain a sizeable equity premium of 4.98% but remains somewhat below the empirically observed value of 5.88%. On the other hand, the return on housing in the model (4.42%) turns out too low compared to its empirical counterpart (6.01%). Combined with the too-large risk-free rate in the model, the model can only generate a housing premium of 1.35% and remains significantly below the 4.45% found in the U.S. data. Accordingly, the total risk premium in the model is also too low compared to the data (3.02% vs. 5.27%).

Moreover, while the risk-free rate is less volatile in the model (standard deviation of 1.40) than in the data (standard deviation of 2.31), the return on equity in the model is almost twice as volatile as in the data (standard deviation of 27.02 in the model vs. 16.71 in the data), and the return on housing is even more than twice as volatile as in the data (standard deviation of 10.40 in the model vs. 3.78 in the data). The standard deviations of the premia are similar to the standard deviations of the return rates and, in consequence, the model fails to explain the observed Sharpe ratios. The Sharpe ratio of equity in the model (0.19) turns out too low by a factor of 2 compared to the data (0.36) and the Sharpe ratio of housing in the model (0.14) remains below the empirical value (1.01) by a factor of even more than 10.

To provide some additional reasoning for the model's failures with regard to asset price statistics, we summarize the (annualized) decomposition of the Sharpe ratios provided by equation (12) in the second row of Table 5. Concerning our fundamental requirements discussed in section 3, the decomposition first reveals that the HJB turns out far too small at 0.18. Although risk premia are perfect negatively correlated with the SDF, the Sharpe ratios in the model remain far too small. Second, the model fails to generate different correlations between the SDF with premia on equity or housing. On the contrary, premia on both assets correlate perfectly , so that their Sharpe ratios are—at odds with the data—identical. Since Sharpe ratios are far too small, the model must rely on too large volatilities

Table 4: Simulated returns, premiums and second moments

	USA	External Habits	Long-Run Risk	Disaster Risk no disaster sample	Disaster Risk disaster sample
Panel A: Business Cycle					
$\sigma(GDP)$	1.52	1.41	1.47	0.92	1.63
$\frac{\sigma(I)}{\sigma(GDP)}$	2.91	1.01	0.69	2.19	1.64
$\frac{\sigma(D)}{\sigma(GDP)}$	6.85	7.44	2.56	6.49	5.10
$\frac{\sigma(P_H)}{\sigma(GDP)}$	2.03	2.33	0.77	1.95	1.53
$\rho(P_H, D)$	0.67	1.00	1.00	1.00	1.00
$\rho(I, D)$	0.07	0.96	0.84	0.97	0.90
$\rho(GDP, D)$	0.72	0.99	0.91	0.68	0.58
$\rho(GDP, P_H)$	0.64	0.99	0.91	0.68	0.58
Panel B: Return Rates					
R_g	1.57	3.04	2.24	1.36	1.12
R_E^{lev}	7.45	8.14	8.31	8.94	7.93
R_H	6.01	4.42	3.20	3.99	3.43
R_T^{lev}	6.84	6.14	5.62	6.43	5.73
EP	5.88	4.98	5.97	7.52	6.76
HP	4.45	1.35	0.94	2.62	2.29
TP	5.27	3.02	3.32	5.03	4.58
$\sigma(R_g)$	2.31	1.40	0.99	1.61	1.75
$\sigma(R_E^{lev})$	16.71	27.02	9.95	13.27	12.67
$\sigma(R_H)$	3.78	10.40	2.43	3.42	4.36
$\sigma(R_T^{lev})$	6.90	18.74	5.81	7.98	8.40
$\sigma(EP)$	16.47	26.09	9.91	13.80	13.03
$\sigma(HP)$	4.41	9.89	2.23	4.09	4.65
$\sigma(TP)$	7.00	18.03	5.73	8.54	8.74
SR_E	0.36	0.19	0.60	0.55	0.52
SR_H	1.01	0.14	0.42	0.64	0.49
SR_T	0.75	0.17	0.58	0.59	0.52

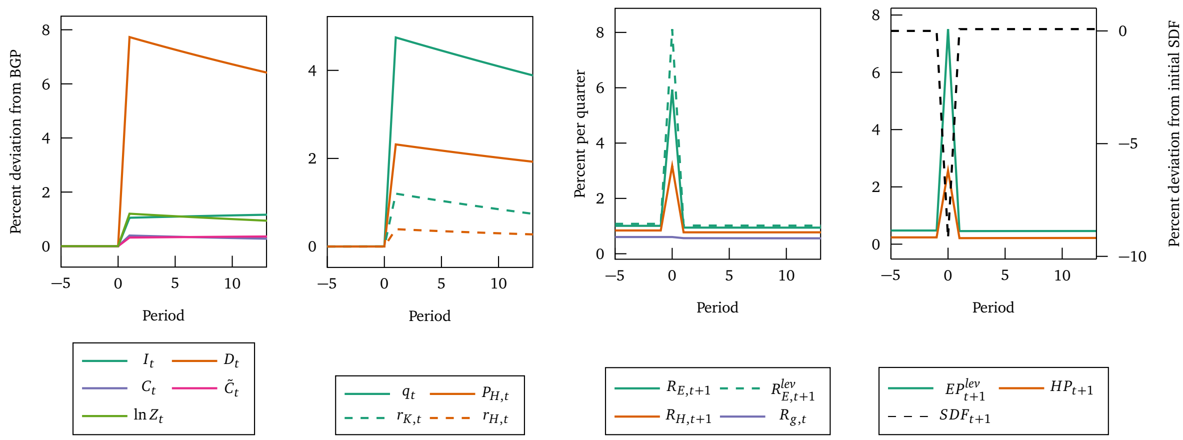
Notes: Business Cycle Moments: Standard deviations $\sigma(.)$ and correlations $\rho(.,.)$ for GDP, business investments I, residential investments D and house prices P_H. Business cycle statistics are reported for logged and HP-filtered (1600) times series.

Return Rates: Mean percentage returns on equity R_E^{lev}, housing R_H, total risk R_T^{lev} and government bonds R_g, as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP), the corresponding standard deviations $\sigma(.)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). All return rates are annualized.

We report the mean outcome from 100 simulations, each for 180 periods after 1000 burn-in periods. For the version with disaster risk, we report the moments from samples without disasters and samples with disasters.

Table 5: Risk Premia: Components

	SR_E	SR_H	$CV(M)$	$\rho(M,EP)$	$\rho(M,HP)$
USA	0.36	1.01	> 1.01	$\in (-0.36;\ 0)$	$2.81 \cdot \rho(M,EP)$
External Habits	0.19	0.14	0.18	−0.97	−0.99
Long-Run Risk	0.60	0.42	0.61	−0.95	−0.64
Disaster Risk					
-no disaster samples	0.55	0.64	0.55	−0.90	−0.75
-disaster samples	0.52	0.49	0.59	−0.85	−0.75



(a) TFP shock $\epsilon_{z,t+1}$ (I) (b) TFP shock $\epsilon_{z,t+1}$ (II) (c) TFP shock $\epsilon_{z,t+1}$ (III) (d) TFP shock $\epsilon_{z,t+1}$ (IV)

Figure 2: Impulse Response Functions for habit formation

of the risky assets to replicate sizeable premia. The counterfactually high volatility of risky returns compensates for the lack of volatility in the SDF.

Finally, we show the impulse response functions to a one-time shock $\epsilon_{z,t+1}$ to total factor productivity Z_{t+1} in Figure 2. The variables' response to the 'classic' technology shock is standard, and business investments, residential investments, and consumption increase in the period the shock hits the economy. Increasing business investments imply an increasing Tobin's q, $q_t = (1/b_2)(I_t/K_t)^\kappa$, and increasing residential investments imply increasing house prices, $P_{H,t} = (1/(1-\varphi))D_t^\varphi$. Although D_t increases more than I_t, the elasticity κ of Tobin's q exceeds the elasticity φ of house prices and Tobin's q expands significantly more than house prices. Moreover, increasing productivity yields an increasing marginal product of capital and increasing consumption implies an increasing marginal rate of substitution between housing and consumption. Consequently, the returns on unlevered equity and housing increase but—mainly due to the higher elasticity of Tobin's q—the return on unlevered equity dominates and leverage further multiplies the effect. By the same token, the (ex-post realized) SDF drops in response to the shock because consumption increases while housing is pre-determined.

The initial response of the SDF to the unanticipated shock deviates from expectations and, therefore, is not reflected in the return on government bonds. This changes in the subsequent periods where expectations meet. Households expect consumption to decline in the following periods from its peak, however, only very slowly. Therefore, the SDF moves only slightly above its initial value, the price of bonds increases very moderately, and the return on government bonds declines only marginally. In the following periods, Tobin's q and house prices decrease from their peak, but the rental rates of capital and housing remain above their initial values. Those effects cancel out, and the return rates of risky assets drop to their initial values, and the same holds for premia.

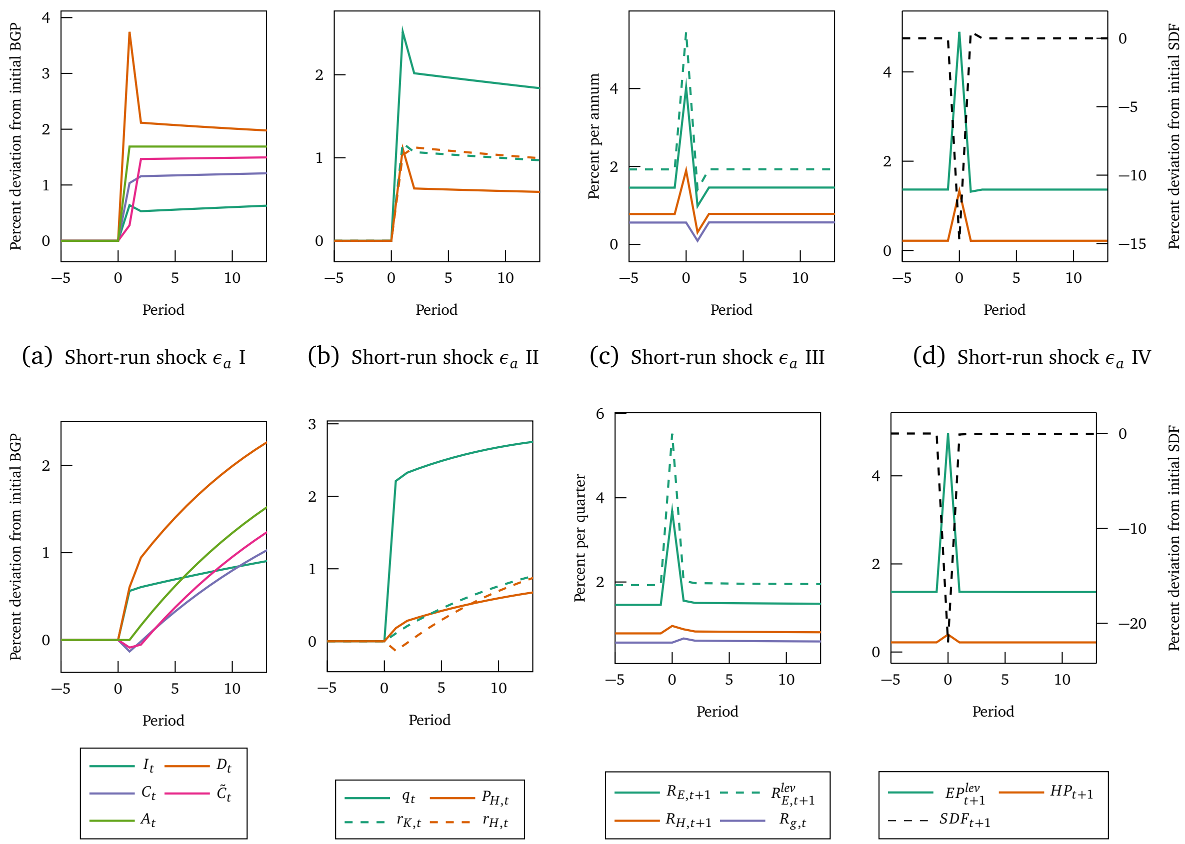
Premia on equity and housing react similarly. They both turn out perfect negatively correlated with the SDF, and the model can not explain different Sharpe ratios. The equity premium in the model turns out higher than the housing premium due to the higher volatility of returns on equity compared to housing. Yet, return rates are already too volatile.

EZ Preferences and Long-Run Risk We summarize the results for the model with external habits in the third column of Table 4. While the model matches the volatility of GDP (1.47 in the model and 1.52 in the data), the model does not generate enough volatility for business investments (0.69 vs. 2.91), residential investments (2.56 vs. 6.85), and house prices (0.77 vs. 2.06).

The risk-free rate, 2.24% in the model, and the return on equity, 8.31% in the model, are both moderately larger than in the data (1.57% and 7.45%, respectively). Hence, the model can closely match the empirically observed equity premium (5.97% in the model and 5.88% in the data). However, the return on housing in the model (3.20%) again turns out too low compared to the value from the data (6.01%). Therefore, the model also fails to explain the housing premium of 4.45% found in the data, as the premium in the model turns out only 0.94%. Consequently, the total risk premium in the model remains also too low compared to the data (3.32% vs. 5.27%).

While the return rates turn out similar to the previous model with external habits, there is a significant difference in the volatility of return rates. Whereas all return rates were too volatile before, volatilities are now too small throughout: the standard deviations of the risk-free rate, the return on equity, and the return on housing turn out 0.99, 9.95, and 2.43, respectively, and therefore remain all below their empirical values of 2.31, 16.71, and 3.78. The volatilities of premia are again close to the volatilities of the return rates. Since the model can replicate the empirical equity premium but generates too involatile return rates, the Sharpe ratio of equity in the model (0.60) exceeds the Sharpe ratio in the data (0.36). The housing premium in the model is by a factor of almost five too low compared to the data, while its standard deviation in the model is only too low by a factor of two. Hence, the Sharpe ratio of housing in the model (0.42) remains too low by a factor of more than 2 compared to the data (1.01). Contrary to the data, the model generates a higher Sharpe ratio for equity than for housing.

The third row of Table 5 summarizes the (annualized) decomposition (12) of the Sharpe ratios for the present variant Compared to the model with external habits, this model generates risk premia in a different way. More specifically, the HJB increases almost by a factor of 4 from 0.16 to 0.61. Premia on equity are still almost perfect negatively correlated with the SDF so that the Sharpe ratio of equity equals the HJB. However, premia on



(a) Short-run shock ϵ_a I (b) Short-run shock ϵ_a II (c) Short-run shock ϵ_a III (d) Short-run shock ϵ_a IV

(e) Long-run shock ϵ_x I (f) Long-run shock ϵ_x II (g) Long-run shock ϵ_x III (h) Long-run shock ϵ_x IV

Figure 3: Impulse Response Functions for Long-run Risk

housing and the SDF are related less linearly. The Sharpe ratio of housing amounts only to approximately 2/3 of the Sharpe ratio of equity with a correlation of -0.64. Unlike the model with external habits, the present model relies on a more volatile SDF that implies larger Sharpe ratios. Premia, therefore, rest on less volatile return rates.

However, the decomposition again reveals two principal failures of the model. First, although the HJB increases substantially compared to the model with external habits, it still remains below the value of 1—the minimum value necessary to allow for a Sharpe ratio as observed for housing in the U.S. data. Second, the correlation between the SDF and premia on housing amounts only to 2/3 of the correlation between the SDF and premia on equity, while the data demands that the former must exceed the latter by a factor of 3. The model's structure separates the correlations between return rates in the wrong direction. The premium on equity rests on a too strong correlation between return rates and the SDF instead of more volatile return rates. On the other hand, the correlation between return rates on housing and the SDF is too small. The resulting Sharpe ratio remains too small and the returns rates do not fluctuate enough so that the premium on housing is considerably smaller than in the data.

We show the effects in response to a shock $\epsilon_{a,t+1}$ to the short-run component of the growth rate in panels (a)-(d) of Figure 3, and the effects of a shock $\epsilon_{x,t+1}$ to the long-run component in panels (e)-(h). The variables' response to the short-run shock are similar to the effects of a TFP shock in Figure 2 for the model with external habits: business investments, residential investments, and consumption increase in the period the shock hits the economy. However, in the present model, consumption increases more, and business and residential investments increase less than in the habit model. As before, residential investments increase significantly more than business investments, but the effect turns for Tobin's q and house prices due to the larger elasticity of Tobin's q. Consequently, returns on equity and housing both increase, but the increase turns out stronger for equity. The (ex-post realized) SDF decreases in the period the shock hits the economy: consumption increases even more than the bundle since housing is fixed and, additionally, lifetime utility increases.

In the next period, business and residential investments drop from their initial peak, which results in a declining Tobin's q and house prices. Although the rental rates of capital and housing remain larger than before the shock, decreasing prices imply that return rates on equity and housing drop below their initial values. Further, consumption increases now less than the composite, since residential investments from the previous period result in more houses. Without unexpected changes to the lifetime utility, this implies that the SDF now increases slightly above its initial value. The price of government bonds increases so that the return on government bonds slightly decreases. Decreasing returns on risky assets and bonds cancel out, and premia return to their initial values.

We now turn to the effects of a shock to the long-run component of the growth rate, which are pictured in panels (e)-(h). This shock has no immediate effect on productivity, but expectations on future productivity increase. Hence, there is an incentive to reallocate output from consumption to investments. While business and residential investments increase, consumption now decreases. Increasing business investments yield an increasing Tobin's q, and the return on equity also rises. However, decreasing consumption and fixed housing imply that the rent of houses decreases. While house prices increase due to more residential investments and higher expected rents, the current drop in rents almost cancels out the former effect. Thus, there is only a tiny increase in the return on housing. Despite consumption declining, the (ex-post realized) SDF drops in response to the shock: lifetime utility increases and is part of the SDF under Epstein and Zin (1989) preferences. The effect on the SDF turns out even stronger in the case of a long-run shock than for a short-run shock, since lifetime utility reacts stronger.

In the subsequent periods, Tobin's q increases only moderately and the return on equity falls close to its initial value. The same holds for house prices and returns on housing. Moreover, consumption increases and, with no unexpected change in the lifetime utility, the SDF remains slightly below its initial value so that the return on government bonds rises marginally above the initial value. Premia essentially return to the values realized before the shock.

EZ Preferences and Disaster Risk Finally, the results for the model with disaster risk are shown in the fourth column of Table 4 for samples that do not include disasters and in the fifth column of Table 4 for samples where we allow disasters in the sample.

Within samples that do not include disasters, the volatility of GDP in the model (0.92) is smaller than in the data (1.52). Moreover, the relative volatilities of business investments (2.19 vs. 2.91), residential investments (6.49 vs. 6.85), and house prices (1.95 vs. 2.03) are all similar to the empirically observed values, yet all moderately smaller. Considering samples that include disasters, the volatility of GDP in the model (1.63) matches the data (1.52) better. However, the relative volatilities of business investments (1.64 vs. 2.91), residential investments (5.10 vs. 6.85), and house prices (1.53 vs. 2.03) turn out smaller now. Residential investments correlate nearly perfectly with both house prices and business investments in both cases. Further, the correlations of residential investments and GDP (0.68 without disasters and 0.58 with disasters) are close to the empirical value (0.72). The same holds for the correlations of house prices and GDP: 0.68 without disasters, 0.58 with disasters, and 0.64 in the data.

All return rates and premia are higher in samples without disasters than in samples that include disasters. In both cases, the model can replicate the return on government bonds fairly well (1.36% without disasters, 1.12% with disasters, and 1.57% in the data). The return on equity exceeds the empirical return by 1.5 percentage points in samples without disasters (8.94% vs. 7.45%) and by 0.5 percentage points in samples that include disasters (7.93% vs. 7.45%). Consequently, the model generates a premium on equity (7.52% without disasters and 6.76% with disasters) somewhat higher than the empirical equity premium (5.88%). On the other hand, the return on housing in the model (3.99% without disasters and 3.43% with disasters) remains below the empirical value (6.01%). Hence, the housing premium in the model (2.62% without disasters and 2.29% with disasters) falls approximately 2 percentage points below the value from the data (4.45%). Compared to the model with long-run risk, the return on government bonds turns out lower, but the return on equity and housing is higher and, consequently, premia on equity and housing are more than 1.5 percentage points higher.

Return rates are moderately more volatile in samples with disasters than without. In both cases, the model can replicate the low volatility of government bonds (1.61 without disasters and 1.75 with disasters) close to the data (2.31). Moreover, return rates on equity are almost as volatile in the model as observed in the data (13.27 without disasters and 12.67 with disasters compared to 16.71 in the data). The same holds for the volatility of equity premia (13.80 without disasters, 13.03 with disasters, and 16.47 in the data). Since premia on equity are higher and less volatile in the model, the Sharpe ratio of equity exceeds the value from the data by a factor of 1.5.

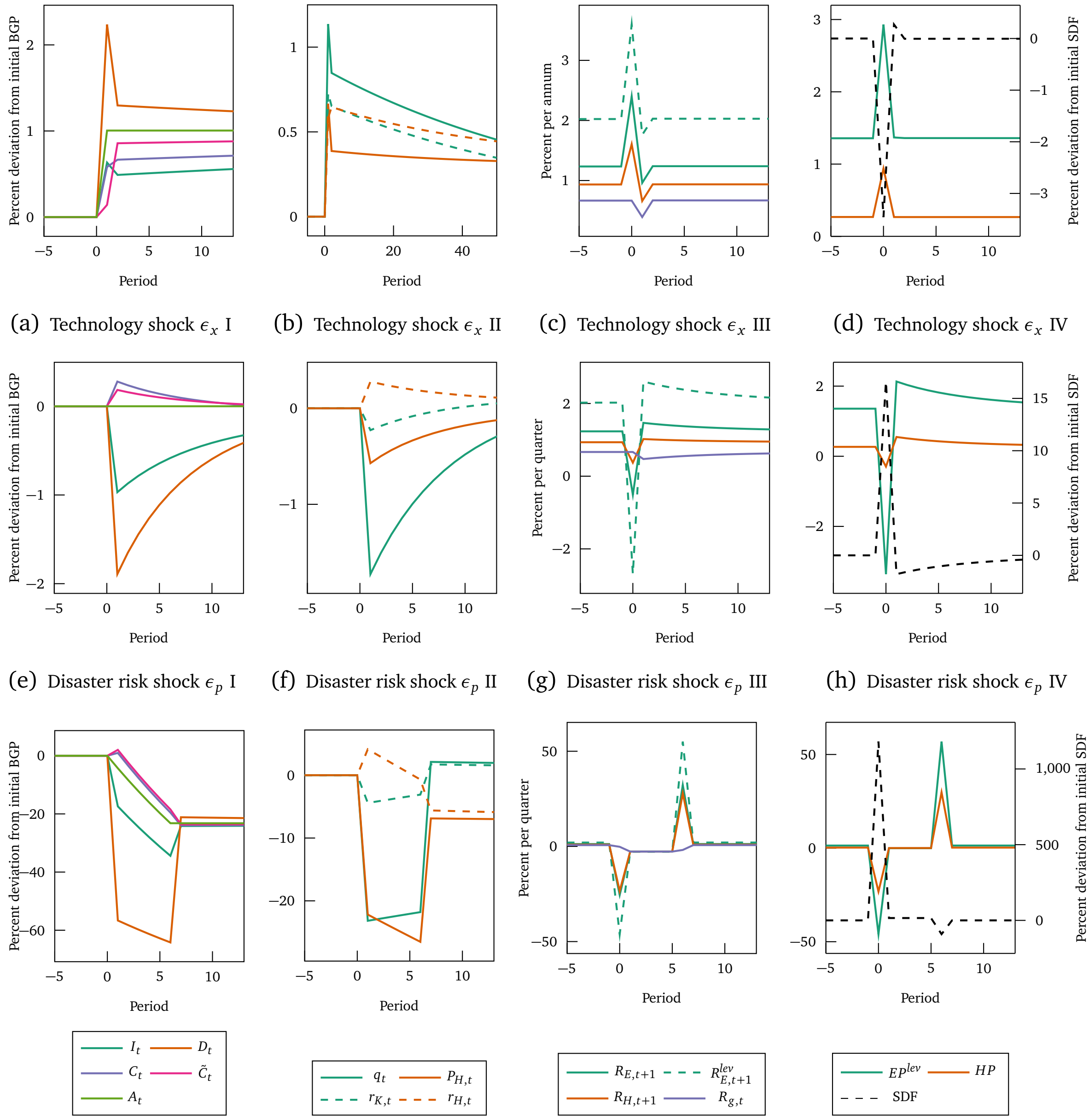
Further, the model can closely match the volatility of returns on housing (3.42 without disasters, 4.36 with disasters compared to 3.78 in the data) as well as the volatility of housing premia (4.09 without disasters, 4.65 with disasters compared to 4.41 in the data). However, since the housing premium in the model remains too low, the Sharpe ratio of housing also turns out too small (0.64 without disasters, 0.49 with disasters, and 1.01 in the data). Compared to the model with long-run risk, the Sharpe ratio of equity remains similar while the Sharpe ratio of housing increases.

Finally, the fourth and fifth rows of Table 5 show again the (annualized) decomposition (12) of the Sharpe ratios. Note, however, that the decomposition only holds for simulations that include disasters. In simulations without disasters, the Lucas (1978) asset pricing equations do not hold on average since the simulated distribution deviates from assumed expectations in the model solution. Compared to the previous model, the coefficient of variation changes only slightly and still falls substantially below the minimum value of approximately 1 – which would be necessary to replicate the empirical Sharpe ratio of housing. While the perfect correlation between the SDF and the equity premium is further dissolved, it remains significantly higher than the upper bound of 1/3 that could potentially allow for a Sharpe ratio of housing that exceeds the Sharpe ratio of equity by a factor of 3. Moreover, although the correlation between the SDF and the housing premium is again raised compared to the previous model, the present model still disentangles the correlations in the wrong way. The correlation between the SDF and premia on housing is smaller than the correlation between the SDF and premia on equity – while the former must exceed the latter by a factor of 3. Consequently, housing's Sharpe ratio turns out slightly smaller than equity's in samples with disasters where expectations meet. The Sharpe ratio of housing can exceed the Sharpe ratio of equity in samples without disasters, which rests on the deviation of the simulated distribution from expectations.

The effects in response to a growth-rate shock $\epsilon_{x,t+1}$, in response to a disaster risk shock $\epsilon_{p,t+1}$, and in response to a disaster shock b_{t+1} are pictured in panels (a)-(d), (e)-(h) and (i)-(l) of Figure 4.[13] First, note that our calibration without autocorrelation of the growth-rate process in normal times, $\rho_x = 0$, implies that the growth-rate shock $\epsilon_{x,t+1}$ acts the same way as the short-run shock in the model with long-run risk. In consequence, the implications of a standard growth-rate shock in the present model turn out almost identical to the effects of a short-run productivity shock from the previous model. The effects in the present model turn out somewhat more moderate since the size of the shock in the present model ($\sigma_x = 0.01$) is smaller than in the model with long-run risk ($\sigma_a = \frac{0.0355}{2}$).

An increase of the probability for the economy to enter a disaster has the following effects (panels (e)-(h)). Positive autocorrelation ($\rho_p > 0$) implies an increased risk for a drop in productivity and destruction of capital and housing in the next period. In consequence, output is shifted from investments in productive capital and from investments in residential structures to consumption. Decreasing investments entail drops in Tobin's q and house prices. The demand effect on house prices and residential investments triggered by the disaster-probability shock is almost identical in size to the effects in response to productivity shocks. Time-varying disaster risk therefore significantly contributes to explain the volatility of housing-related prices and quantities that business cycle models typically fail to replicate. Similar to the previous models, investments in residential structures react stronger than investments in productive capital. Yet, the different elasticities again imply that the effect on Tobin's q dominates the effect on house prices. Moreover, a reduction of working hours implies a decreasing marginal product of capital $r_{K,t}$, whereas increasing consumption implies that the rent $r_{H,t}$ of houses increases. The more pronounced drop in Tobin's q compared to the drop in house prices combined with an increasing rent $r_{H,t}$

[13] A shock to the disaster size does not trigger any effects in the economy in normal times. If there is currently no disaster, the diasters size is irrelevant in the present period and—without autocorrelation, $\rho_\omega = 0$—the present disaster size neither affects expectations about potential future disaster sizes. Therefore, we do not picture the impulse response functions to a disaster-size shock.



(a) Technology shock ϵ_x I (b) Technology shock ϵ_x II (c) Technology shock ϵ_x III (d) Technology shock ϵ_x IV

(e) Disaster risk shock ϵ_p I (f) Disaster risk shock ϵ_p II (g) Disaster risk shock ϵ_p III (h) Disaster risk shock ϵ_p IV

(i) Disaster $b_t = 1$ I (j) Disaster $b_t = 1$ II (k) Disaster $b_t = 1$ III (l) Disaster $b_t = 1$ IV

Figure 4: Impulse Response Functions for Disaster Risk

of houses yields a larger contraction of the return on unlevered equity than of the return on housing and the effect is further amplified by leverage. Finally, increased disaster risk lowers the household's expected future utility and therefore its lifetime utility. Despite the present increase of the consumption bundle, the realized SDF rises substantially under Epstein and Zin (1989) preferences.

In the subsequent periods, the risk for a disaster declines towards its initial value, and, without any actual disaster unfolding, business investments and residential investments begin to increase again from the initial bottom. In consequence, Tobin's q and house prices also increase in the following periods, and return rates on risky assets raise slightly above the rates before the shock. Further, the still increased risk for disasters causes bond prices to increase so that the return on the government bond declines. Premia raise above their initial values. On the other hand, the fact that no disaster occurs despite the increased risk improves the household's lifetime-utility relative to expectations and lowers the ex-post realized SDF below the initial value.

Lastly, an occurrence of a disaster (panels ((i)-(l)) implies that technology A_t drops by the factor $e^{\bar{\omega}}$ as long as the disaster continues. In the period the disaster starts, a second effect appears. The probability that the disaster remains persistent raises to $q = 0.91$, whereas the probability to enter a disaster was initially only $p \approx 0.0072$. The massive increase in probability for the continued destruction of technology, capital, and residential structures in the subsequent period has the previously described effects—amplified by a multitude. The two effects combined—drop in productivity and increased risk for the disaster to persist—cause huge drops in business investments and residential investments in the initial period of the disaster. In the following disaster periods, expectations do not change anymore until the disaster ends so that investments are only affected by decreasing technology, capital, and residential structures. The initial drop of business investments exceeds the destruction of productive capital so that Tobin's q also collapses. In the following periods, the effect turns and business investments decline by less than the rate at which capital is destructed so that Tobin's q begins to slowly recover. On the other hand, since land is not destructed, house prices continue to decline as long as D_t declines. Finally, once the disaster ends, the probability for the economy to be hit by a disaster again jumps back to $p \approx 0.0072$. The massive change in expectations leads to a boom immediately after the disaster. Both investments increase and so do Tobin's q and house prices. The huge drops in Tobin's q and house prices at the start of the disaster yield huge drops in the return rates while the boom after the disaster ends implies huge yields of both risky assets. The realized SDF reacts the opposite way.

5.3 Discussion

Finally, we briefly discuss our conclusion as for why the present framework with productivity risk only fails to replicate the observed Sharpe ratios in their relative size. While we argue that different specifications of the composite good or increased technological frictions do not improve the model's fit, the section concludes with further thoughts on mechanisms that may help.

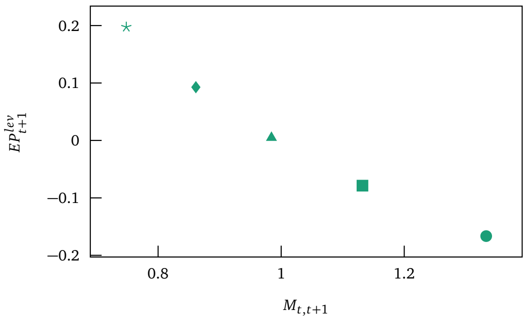
Insufficient explanatory power of technology shocks: The results from the preceding section revealed two failures concerning asset prices that are common to our framework: i) the relation between returns on stocks and the SDF turns out to be too linear, and ii) the relation is, contrary to what is required, less linear between returns on housing and the SDF. This result is further illustrated in Figure 5 which shows scatter plots of these variables. More specifically, we numerically compute expectations by Gauss-Hermite quadrature with five nodes for each shock. Conditional on the state in period t being on the deterministic growth path, the figure shows the corresponding realizations of the tuples $(M_{t,t+1}, EP^{lev}_{t+1})$ and $(M_{t,t+1}, HP_{t+1})$ at these nodes.

The first row of the figure pictures the scatter plots for the model with external habits. The marks distinguish between different realizations of the total factor productivity. As can bee seen, equity and housing premia are strongly negatively and linearly related to the SDF. Hence, their correlations with the SDF are close to -1, and, consequently, the model predicts similar Sharpe ratios of both assets.

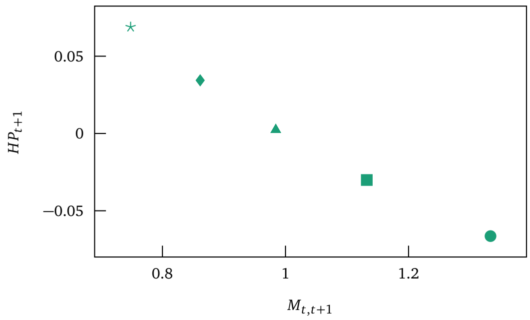
The second row of the figure pictures the scatter plots for the model with long-run risk. Different marks distinguish between different realizations of the short-run risk component and, different colors identify the long-run risk component. The plot on the left-hand side shows that the effect on the return on equity relative to the effect on the SDF is similar between the two shocks (see also Figure 3 for the details). Consequently, there is a strong linear relationship between the two variables and, the correlation is close to -1.[14] The plot on the right-hand side reveals that a long-run productivity shock has significantly less effects on returns on housing than a short-run productivity shock, while the opposite is true for the effects on the SDF (see again Figure 3 for the details). Hence, the effect on the return on housing relative to the effect on the SDF differs significantly between the two shocks. Although there is a strong linear relation between $M_{t,t+1}$ and HP_{t+1} for each shock individually when keeping the other shock constant, the slopes are substantially different. The relation becomes more scattered with a less perfect negative correlation.

Finally, the scatter plots for the model with disaster risk are shown in the third and fourth row of the figure. If no disaster occurs, the disaster size shock is irrelevant and therefore not pictured. In this case, different marks distinguish between particular realizations of the technology shock, while different colors identify the disaster risk. In case a disaster occurs, the disaster size shock becomes active and, we abstain from distinguishing between the then 125 shock realizations by color or marks. The picture on the left-hand side now shows that the technology shock has larger effects on the return on equity compared the disaster risk shock, while the disaster risk shock has larger effects on the SDF than the technology shock (see also Figure 4). While the relation between returns on stocks and the SDF is again highly linear in each shock individually, the slopes turn out somewhat more different between the two shocks and, the high (negative) correlation can be moderately reduced. However, the relation remains far too linear. Further, the plot on the right-hand side shows that the effect of a disaster risk shock on the return on housing is even smaller (see again Figure 4). As in the model with long-run risk, the slopes in the relation between

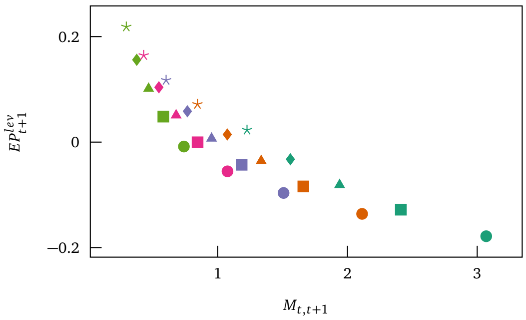
[14]Note that while the return on equity and the SDF may be non-linear in the shocks, they display similar non-linearities, and the relation between the two variables turns out linear again.



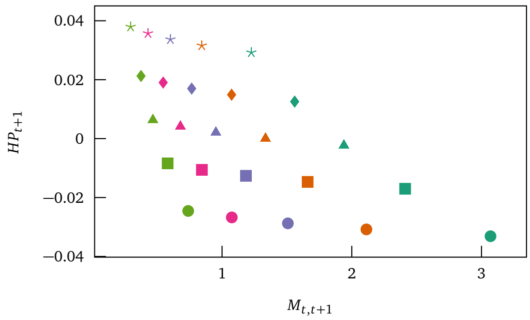
(a) External Habits: Equity



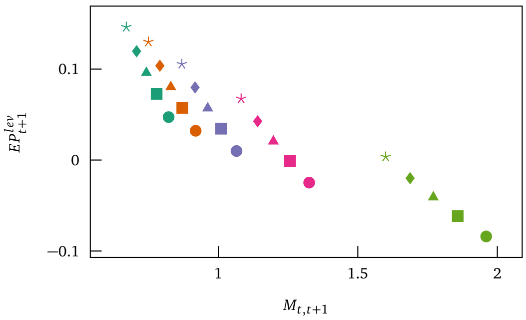
(b) External Habits: Housing



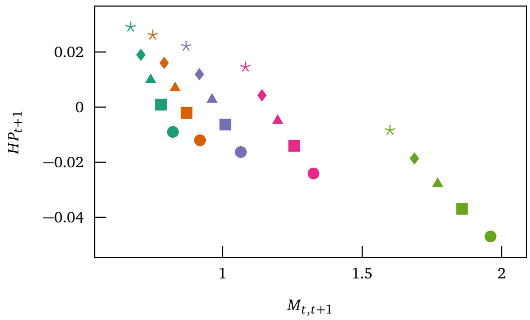
(c) Long-Run Risk: Equity



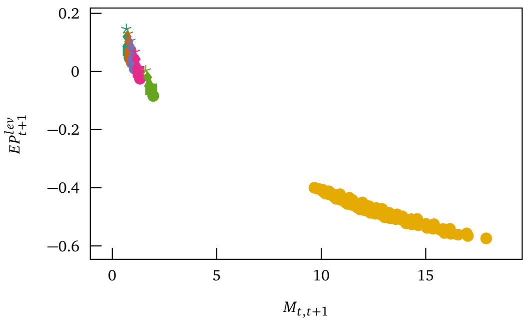
(d) Long-Run Risk: Housing



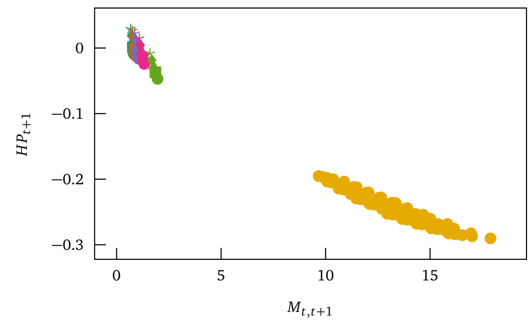
(e) Disaster Risk: Equity (no disaster)



(f) Disaster Risk: Housing (no disaster)



(g) Disaster Risk: Equity (disaster)



(h) Disaster Risk: Housing (disaster)

Figure 5: Correlation: Returns and SDF

returns on housing and the SDF turn out more differently. Contrary to what is required, returns on housing are correlated less with the SDF.

Summing up, a possible first step to simultaneously explain premia on equity and housing could be to invert the effects seen for the model with long-run productivity risk or disaster risk. This would require two shocks: the first with larger effects on the SDF but less effects on the return on equity, while the second shock affects the SDF less but has larger effects on the returns on stocks. Additionally, the relative effects for returns on housing and the SDF need to be similar between the two shocks. We could not meet these requirements with productivity risk.

Specification of the composite good: While the literature concludes about non-separable preferences over housing and non-durable consumption, there is less consensus about the intratemporal elasticity of substitution between both goods, which is why we choose an intratemporal constant elasticity of substitution between housing and non-durable consumption of one. Further, by varying the intratemporal elasticity, we could not improve the model's fit. Increasing the substitutability would reduce the already too-low housing premia. Reducing the intratemporal substitutability would increase risk premia, but at the cost of the demand effect for housing. Since the demand effect may track the business cycle statistics, an increase in housing premiums comes with insufficient volatility in house prices and residential investments.

Another point to discuss relates solely to habit formations. In our specification, the habits are superficial, i.e., the formation is over the composite good $\tilde{C}_t$. Another common specification is called deep habits, where the formation is related to the individual goods C_t and H_t. Here, this would compromise the household's possibility to substitute housing for consumption, which could improve asset price statistics. We checked the ability, while the asset price statistics scarcely improve, the volatility of GDP nearly doubles as the rental rate of housing becomes excessively volatile.

Lastly, neither changing intratemporal substitutability nor the habit formation allow replicating a housing Sharpe ratio surplus.

Technological restrictions: In our framework the marginal rate of transformation between residential investment and consumption is one. The literature on housing and the business cycle restricts this possibility often. Unfortunately, this could also not improve the model's fit. Although a concave productivity possibility frontier limits the household to smooth its composite good by substituting housing for consumption and thus, would increase housing premia. However, the limited substitutability would also reduce the volatility of residential investment and, thereby, limit the demand effect for housing contrary to the data. Concerning the too-low business investment volatility, the productivity possibility frontier is too strict. Loosening this restriction would not improve the business cycle statistics as with the lower reward for holding risk, procyclical demand for housing vanishes.

Again, neither changes in the productivity possibility frontier of residential nor business investment allow replicating a housing Sharpe ratio surplus.

Further thoughts: The principal deficit of the model is obvious. The return rates and also the premia of equity correlate almost perfectly with the SDF. Any mechanism that increases the volatility of the return on equity and decreases, in absolute terms, the correlation of the return on equity with the SDF would improve the model's fit.

Assuming that cooperate bonds could additionally default in normal times meets these requirements.[15] The additional source of uncertainty increases the volatility of the return on equity while the assumption of independence decreases in absolute terms the correlation between the SDF and the return on equity.

Other mechanisms concerning housing specific characteristics could improve the model's fit in general. For example, due to the poor divisibility of housing, there may be credit-constrained households who can only invest in equity. For them, it would be impossible to smooth the consumption bundle by adjusting consumption and residential investment and subsequently, the equity risk would increase. Albeit, housing investment participation distributes far broader and is less concentrated towards the top quantiles than the participation at the equity stock market as Kuhn et al. (2020) show. This participation distribution indicates that the effect is minimal at best.

Among others, Mian et al. (2013) find a strong effect of housing wealth on consumption. Modeling such a channel would increase in absolute terms the correlation between house prices and the SDF and thus between the return on housing and the SDF, which would separate the Sharpe ratios. Theoretical foundations for strong causal effects are given, e.g., by Berger et al. (2017) and Guerrieri and Lorenzoni (2017). Gertler and Gilchrist (2018) summarize this channel in a review as follows: Mortgages are the household's most common structure of debt. Hence, declining house prices increase the households leverage ratio and the resulting tightened (re)financing options force the household to reduce its consumption spending. However, Khan and Rouillard (2018) show that household borrowing constraints alone cannot account for house prices volatility.

Last, the risk on housing wealth is potentially more idiosyncratic, which increases the volatility of the return on housing on an individual level and thus helps to explain differences in the Sharpe ratios at the aggregated level. However, we argued that other determinants must also be at work and that the main shortcoming of the model is the almost perfect correlation of the SDF with equity premia. For this reason, we agree with Jordà et al. (2019) and conclude that a putative solution via idiosyncratic housing falls short.

6 Conclusion

We confront existing approaches to solve the equity premium puzzle with the presence of a second asset, namely housing, and the data from Jordà et al. (2019) on returns on equity, housing, and total wealth. Our framework is a standard RBC model with housing that features different types of productivity risk: standard productivity risk, long-run productivity risk as in Croce (2014), or disaster risk similar to Gourio (2012). The stochastic

[15] Gourio (2012) argues e.g. the financial crisis 2008 was not a great disaster and US-treasury bonds and bills did not default. Nevertheless, a lot of corporate bonds defaulted.

discount factor follows either from preferences with external habits as in Chen (2017) or from generalized recursive Epstein and Zin (1989) preferences.

The main results of our study with regards to asset pricing statistics are as follows. First, the models retain their previously documented ability to generate sizeable equity premia with the introduction of housing as a second asset. Second, as in the data, housing premia in all models are smaller than premia on eqity. Yet, while there is only a small difference between the premia of the two assets in the data, housing premia in the model are too small. Third, in all models considered, the mechanism to generate sizeable premia on eqity relies on a far too high correlation between returns on eqity and the marginal utility of consumption. The high correlation already excludes a second asset with a significantly larger Sharpe ratio, as empirically observed for housing. Fourth, the mechanism relies insufficiently on a sizeable HJB. This quantity remains too small to explain the size of the Sharpe ratio of housing. Fifth, returns on housing correlate less than equity with the marginal utility of consumption. Contrary to the data, the Sharpe ratio of housing in the model falls below the Sharpe ratio of equity.

Additionally, we examine the model's ability to reproduce business cycle statistics. The model with standard productivity risk and external habits as well as the modelith disaster risk can replicate the volatilities of GDP, business investments, residential investments, and house prices. The ability to replicate the volatility of business investments in the model with external habits comes at the expense of the ability to replicate asset statistics. In the model with long-run productivity risk, the volatility of business investments, residential investments, and house prices remains too low. The model with disaster risk can further explain the empirically observed correlations between GDP and residential investments and between GDP and house prices. Otherwise, the correlations are only matched in sign but turn out too large.

The high correlation of equity returns and the SDF is the weak point in the explanatory power of all variants in accounting for different Sharpe ratios and risk premia. Disentangling this relationship points the way for future research.

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APPENDIX
(not for publication)

The return on everything and the business cycle in production economies

A Stylized facts and data Resources

A.1 Stylized facts

In Table 6 we present return rates for all countries from the JKKST database. We observe the following stylized facts. First, risk premia in all countries are sizeable with equity premia between 1.17 percent in Portugal and 12.91 percent in Finland, and housing premia between 3.47 and 8.39 percent in Germany and Norway, respectively. Second, in all countries except for Italy and Portugal, the return on housing is lower than the return on equity. Third, in all countries listed the volatility of returns and premia on equity exceeds the volatility of returns and premia on housing and the volatility of the risk-free rate is the smallest. Fourth, the Sharp ratio of housing is larger than of equity in all listed countries. Last, there is no systematic nexus between the return on housing and equity.

Table 7 displays the business cycle statistics for the same countries. Note that for several continental European countries the standard deviation of residential investment exceeds the standard deviation of business investment only slightly or is even smaller. More precisely, this is the case for France, Finland, Germany, Italy, the Netherlands, Norway, Portugal, and Spain. House prices are pro-cyclical with GDP and more volatile than GDP in all countries except for Germany. Business investment and residential investment are positively correlated in all countries but Sweden and Australia, and house prices and residential investment are positively correlated throughout.

A.2 Sources

The data pertaining the rates of return, mortgage etc. are from JKKST. The source of the data pertaining the business cycle statistics and the homeownershiprate is listed below:

- GDP, residential investment, non-residential investment: OECD Economic Outlook Nov 2018; Denmark: Statistics Denmark.
- House prices: OECD Real house price indices, s.a. 16.05.2019 devided by the OECD Economic Outlook Nov 2018 CPI Deflator.
- Population: FRA, USA: OECD Total population PERSA: Persons, seasonally adjusted; UK: Office for National Statistics UK, resident population: mid-year estimates (Qtly data interpolated (by the Office); Otherwise: Yearly, Worldbank, midyear (interpolated (by own calculation)).
- Homeownershiprates: Japan (2007): http://www.stat.go.jp/english/index.html; USA (2007), Australia (2003): https://www.oecd.org/eco/growth/evolution

  %20of%20homeownership%20rates.pdf p.212; Otherwise (2010): EUROSTAT "Eurostat - Data Explorer - Distribution of population by tenure status, type of household and income group"

Table 6: Returns, premiums and second moments

	R_E	R_H	R_T	R_g	EP	HP	TP
AUS	6.84	7.01	7.22	2.00	4.84	5.02	5.22
BEL	10.29	8.02	8.65	2.49	7.80	5.53	6.16
DNK	13.35	6.6	8.69	0.72	12.63	5.88	7.96
FIN	14.17	8.84	11.67	1.25	12.91	7.59	10.41
FRA	9.61	5.78	6.61	2.24	7.37	3.54	4.37
GER	9.04	4.75	5.75	1.28	7.76	3.47	4.47
ITA	4.76	5.62	5.89	1.55	3.21	4.07	4.34
JPA	5.86	5.54	6.19	0.98	4.88	4.56	5.21
NLD	8.93	7.83	8.12	1.44	7.49	6.38	6.68
NOR	13.03	9.82	10.66	1.43	11.60	8.39	9.23
PRT	2.92	6.65	6.62	1.75	1.17	4.89	4.87
ESP	7.10	4.43	4.89	0.69	6.40	3.74	4.20
SWE	12.29	8.82	10.56	0.91	11.38	7.92	9.65
CH	7.23	6.01	6.98	−0.17	7.40	6.19	7.16
UK	8.00	7.00	7.47	1.56	6.44	5.44	5.91
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_g)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$
AUS	21.53	5.71	6.02	3.32	21.32	6.22	6.27
BEL	22.99	6.04	6.37	2.84	23.08	6.61	7.02
DNK	23.84	7.72	8.91	1.56	24.33	7.24	8.93
FIN	37.72	9.13	21.58	4.52	36.90	9.64	21.39
FRA	24.11	5.52	6.95	2.55	23.98	6.18	7.39
GER	22.82	3.12	5.09	1.73	23.12	4.30	5.99
ITA	27.98	10.77	10.07	3.18	27.57	11.51	10.74
JPA	20.15	6.53	8.10	2.53	19.94	6.47	8.03
NLD	22.06	9.14	9.40	2.91	22.13	9.68	9.91
NOR	29.60	8.51	9.02	2.39	29.66	9.64	10.12
PRT	26.86	7.26	7.64	2.57	26.81	7.39	7.64
ESP	27.13	8.36	8.62	4.43	25.93	8.19	8.10
SWE	26.03	7.35	11.67	2.06	25.75	7.22	11.42
CH	21.61	4.59	8.01	2.29	21.41	4.94	7.97
UK	23.41	9.64	8.44	3.73	24.27	8.88	8.62
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00
	SR_E	SR_H	SR_T				
AUS	0.23	0.81	0.83				
BEL	0.34	0.84	0.88				
DNK	0.52	0.81	0.89				
FIN	0.35	0.79	0.49				
FRA	0.31	0.57	0.59				
GER	0.34	0.81	0.75				
ITA	0.12	0.35	0.4				
JPA	0.24	0.7	0.65				
NLD	0.34	0.66	0.67				
NOR	0.39	0.87	0.91				
PRT	0.04	0.66	0.64				
ESP	0.25	0.46	0.52				
SWE	0.44	1.1	0.85				
CH	0.35	1.25	0.9				
UK	0.27	0.61	0.69				
USA	0.36	1.01	0.75				

Notes: Mean percentage returns on equity R_E^{lev}, housing R_H, total risk R_T^{lev} and government bonds R_g, as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP), the corresponding standard deviations $\sigma(.)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). Asset price statistics are computed for annual data. Source: JKKST, own calculations Periods: Australia 1970-2015, Belgium 1976-2015, Denmark 1995-2015, Finland 1970-2015, France 1980-2015, Germany 1991-2015, Italy 1970-2015, Japan 1963-2015, the Netherlands 1970-2015, Norway 1978-2015, Portugal 1988-2015. Spain 1971-2015, Sweden 1970-2015, Switzerland 1970-2015, United Kingdom 1969-2015, USA 1970-2015, data from JKKST, own calculations.

Table 7: Empirical business cycle statistics

	$\sigma(GDP)$	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$\rho(P_H, D)$	$\rho(I, D)$	$\rho(GDP, D)$	$\rho(GDP, P_H)$
AUS	1.24	3.77	6.65	3.56	0.55	−0.12	0.56	0.35
BEL	1.02	4.21	7.19	3.54	0.62	0.22	0.47	0.31
DNK	1.42	3.52	5.95	3.96	0.60	0.28	0.66	0.75
FIN	2.21	2.87	3.15	3.00	0.73	0.40	0.67	0.66
FRA	0.95	2.75	3.17	3.19	0.65	0.64	0.81	0.48
GER	1.47	2.54	2.20	0.82	0.06	0.49	0.57	−0.16
ITA	1.44	2.66	1.67	3.73	0.25	0.37	0.46	0.15
JPA	1.59	2.41	3.84	2.70	0.31	0.27	0.45	0.55
NLD	1.37	5.86	5.57	4.08	0.40	0.31	0.48	0.35
NOR	1.46	4.68	4.51	3.72	0.60	0.28	0.31	0.56
PRT	1.58	3.37	2.58	1.87	0.40	0.56	0.64	0.50
ESP	1.33	3.48	3.42	4.17	0.43	0.69	0.77	0.61
SWE	1.51	4.46	5.20	2.95	0.42	−0.22	0.04	0.57
CHE	1.64	–	–	2.69	–	–	–	0.61
UK	1.58	2.68	5.56	4.85	0.51	0.16	0.69	0.71
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64

Notes: Business Cycle Moments: Standard deviations $\sigma(.)$ and correlations $\rho(.,.)$ for GDP, business investments I, residential investments D and house prices P_H. Business cycle statistics are computed for logged and hp-filtered (1600) quarterly per-capita data. Periods: Australia 1970-2015, Belgium 1976-2015, Denmark 1995-2015, Finland 1970-2015, France 1980-2015, Germany 1991-2015, Italy 1970-2015, Japan 1963-2015, the Netherlands 1970-2015, Norway 1978-2015, Portugal 1988-2015. Spain 1971-2015, Sweden 1970-2015, Switzerland 1970-2015, United Kingdom 1969-2015, USA 1970-2015, Data: OECD.stats, own calculations.

B Models' systems of equation and solution

B.1 External habits

General Equilibrium Summing up, in any period t the economy's equilibrium is characterized by the following system of equations

$$Y_t = Z_t K_t^{\alpha} (A_t)^{1-\alpha}, \tag{15a}$$

$$r_{K,t} = \alpha \frac{Y_t}{K_t}, \tag{15b}$$

$$Y_t = C_t + I_t + D_t, \tag{15c}$$

$$\tilde{C}_t = C_t^{\mu_C} (A_{t-1}^{\varphi} H_t)^{\mu_H} \tag{15d}$$

$$P_{H,t} = \frac{1}{1-\varphi} D_t^{\varphi}, \tag{15e}$$

$$q_t = \frac{1}{b_2} \left(\frac{I_t}{K_t} \right)^{\kappa}, \tag{15f}$$

$$K_{t+1} = (1-\delta_K) K_t + \Phi \left(\frac{I_t}{K_t} \right) K_t, \tag{15g}$$

$$H_{t+1} = (1-\delta_H) H_t + D_t^{1-\varphi}, \tag{15h}$$

$$\ln\left(\frac{\tilde{C}_{t+1}-\tilde{C}_{h,t+1}}{\tilde{C}_{t+1}}\right)=(1-\rho_{\tilde{C}})\ln(\bar{S}_{\tilde{C}})+\rho_{\tilde{C}}\ln\left(\frac{\tilde{C}_t-\tilde{C}_{h,t}}{\tilde{C}_t}\right)+\left(\frac{1}{\bar{S}_{\tilde{C}}}-1\right)\left(\ln\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)-a\right), \tag{15i}$$

$$q_t=\mathbb{E}_t\left[M_{t,t+1}\left(r_{K,t+1}-\frac{I_{t+1}}{K_{t+1}}+q_{t+1}\left(1-\delta_K+\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right)\right], \tag{15j}$$

$$P_{H,t}=\mathbb{E}_t\left[M_{t,t+1}\left(\frac{\mu_H}{\mu_C}\frac{C_{t+1}}{H_{t+1}}+P_{H,t+1}(1-\delta_H)\right)\right], \tag{15k}$$

$$\tag{15l}$$

given the state variables $K_t, H_t, \tilde{C}_{ht}, Z_t, A_t$. The SDF satisfies

$$M_{t,t+1}=\beta\left(\frac{C_{t+1}}{C_t}\right)^{\mu_C-1}\left(\frac{H_{t+1}}{H_t}\right)^{\mu_H}\left(\frac{\tilde{C}_{t+1}-\tilde{C}_{h,t+1}}{\tilde{C}_t-\tilde{C}_{ht}}\right)^{-\gamma}.$$

Moreover, the exogenous variables Z_t and A_t are governed by the processes

$$\ln\left(\frac{A_{t+1}}{A_t}\right)=a, \tag{16a}$$

$$\ln Z_t=\rho_z\ln Z_{t-1}+\epsilon_{z,t},\ \epsilon_{z,t}\sim iid\mathcal{N}(0,\sigma_z^2), \tag{16b}$$

Finally, we define GDP as the sum of consumption, both investment types and the implicit rent from housing

$$\text{GDP}_t=Y_t+r_{H,t}H_t,\ \ \text{where } r_{H,t}:=\frac{\mu_H}{\mu_C}\frac{C_t}{H_t}.$$

We scale the variables in terms of technology A_t by $k_t:=\frac{K_t}{A_t}$, $h_t:=\frac{H_t}{A_t^{1-\varphi}}$, $y_t:=\frac{Y_t}{A_t}$, $c_t:=\frac{C_t}{A_t}$, $i_t:=\frac{I_t}{A_t}$, $d_t:=\frac{D_t}{A_t}$, $p_{H,t}:=\frac{P_{H,t}}{A_{t-1}^{\varphi}}$, $\tilde{c}_t:=e^{\mu_H\varphi a}\frac{\tilde{C}_t}{A_t}$, and $\tilde{c}_{ht}:=e^{\mu_H\varphi a}\frac{\tilde{C}_{ht}}{A_t}$. Hence, the system of equations (15) and 16 can be written equivalently in terms of the scaled variables as

$$y_t=Z_tk_t^{\alpha}, \tag{17a}$$

$$r_{K,t}=\alpha\frac{y_t}{k_t}, \tag{17b}$$

$$y_t=c_t+i_t+d_t, \tag{17c}$$

$$\tilde{c}_t=c_t^{\mu_C}h_t^{\mu_H}, \tag{17d}$$

$$p_{H,t}=\frac{1}{1-\varphi}d_t^{\varphi}, \tag{17e}$$

$$q_t=\frac{1}{b_2}\left(\frac{i_t}{k_t}\right)^{\kappa}, \tag{17f}$$

$$e^ak_{t+1}=(1-\delta_K)k_t+\Phi\left(\frac{i_t}{k_t}\right)k_t, \tag{17g}$$

$$e^{a(1-\varphi)}h_{t+1}=(1-\delta_H)h_t+d_t^{1-\varphi} \tag{17h}$$

$$\ln\left(\frac{\tilde{c}_{t+1}-\tilde{c}_{h,t+1}}{\tilde{c}_{t+1}}\right)=(1-\rho_{\tilde{C}})\ln(\bar{S}_{\tilde{C}})+\rho_{\tilde{C}}\ln\left(\frac{\tilde{c}_t-\tilde{c}_{h,t}}{\tilde{c}_t}\right)+\left(\frac{1}{\bar{S}_{\tilde{C}}}-1\right)\ln\left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right), \tag{17i}$$

$$q_t=\mathbb{E}_t\left[e^{-\gamma a}m_{t,t+1}\left(r_{K,t+1}-\frac{i_{t+1}}{k_{t+1}}+q_{t+1}\left(1-\delta_K+\Phi\left(\frac{i_t}{k_t}\right)\right)\right)\right], \tag{17j}$$

$$p_{H,t}=\mathbb{E}_t\left[e^{(-\gamma+\varphi)a}m_{t,t+1}\left(\frac{\mu_H}{\mu_C}\frac{c_{t+1}}{h_{t+1}}+p_{H,t+1}(1-\delta_H)\right)\right], \tag{17k}$$

where

$$m_{t,t+1}=\beta\left(\frac{c_{t+1}}{c_t}\right)^{\mu_C-1}\left(\frac{h_{t+1}}{h_t}\right)^{\mu_H}\left(\frac{\tilde{c}_{t+1}-\tilde{c}_{h,t+1}}{\tilde{c}_t-\tilde{c}_{ht}}\right)^{-\gamma}.$$

Solution Method First, note that given period t's scaled state variables k_t, h_t, Z_t, and $\left(\frac{\tilde{c}_t-\tilde{c}_{h,t}}{\tilde{c}_t}\right)$ and the control variables for residential investment d_t and consumption c_t, all other period t variables as well as next period's endogenous state variables can be easily computed from equations (17a)-(17i). We approximate the policy functions for c_t and d_t by linear combinations of Chebyshev polynomials. We compute the coefficients in the linear combinations such way that the Euler equations (17j) and (17k) are satisfied exactly at a sparse grid of collocation points (see Judd et al. (2014) and Heer and Maussner (2009) for details). Thereby, the expectations with respect to normally distributed random variables are computed by Gauss-Hermite quadrature.

B.2 Long-run risk

General Equilibrium Summing up, in any period t the economy's equilibrium is characterized by the following system of equations

$$Y_t=K_t^{\alpha}(A_tN_t)^{1-\alpha}, \tag{18a}$$

$$r_{K,t}=\alpha\frac{Y_t}{K_t}, \tag{18b}$$

$$W_t=(1-\alpha)\frac{Y_t}{N_t}, \tag{18c}$$

$$W_t=\frac{\mu_N}{\mu_C}\frac{C_t}{1-N_t}, \tag{18d}$$

$$Y_t=C_t+I_t+D_t, \tag{18e}$$

$$\tilde{C}_t=C_t^{\mu_C}(A_{t-1}^{\varphi}H_t)^{\mu_H}(A_{t-1}(1-N_t))^{\mu_N}, \tag{18f}$$

$$P_{H,t}=\frac{1}{1-\varphi}D_t^{\varphi}, \tag{18g}$$

$$q_t=\frac{1}{b_2}\left(\frac{I_t}{K_t}\right)^{\kappa}, \tag{18h}$$

$$K_{t+1}=(1-\delta_K)K_t+\Phi\left(\frac{I_t}{K_t}\right)K_t, \tag{18i}$$

$$H_{t+1} = (1-\delta_H)H_t + D_t^{1-\varphi}, \tag{18j}$$

$$q_t = \mathbb{E}_t\left[M_{t,t+1}\left(r_{K,t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\left(1-\delta_K + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right)\right], \tag{18k}$$

$$P_{H,t} = \mathbb{E}_t\left[M_{t,t+1}\left(\frac{\mu_H}{\mu_C}\frac{C_{t+1}}{H_{t+1}} + P_{H,t+1}(1-\delta_H)\right)\right], \tag{18l}$$

$$V_t = (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \tag{18m}$$

given the state variables K_t, H_t, A_t. The SDF satisfies

$$M_{t,t+1} = \beta\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{1-\frac{1}{\psi}} \frac{C_t}{C_{t+1}}\left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta}$$

Moreover, A_t follows the stochastic process

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = a + x_t + \epsilon_{a,t+1}, \tag{19a}$$

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t+1}, \tag{19b}$$

where

$$\begin{pmatrix}\epsilon_{a,t+1}\\ \epsilon_{x,t+1}\end{pmatrix} \sim iid\mathcal{N}(0,\Sigma), \quad \text{and } \Sigma = \begin{pmatrix}\sigma_a^2 & \rho_{a,x}\sigma_a\sigma_x\\ \rho_{a,x}\sigma_a\sigma_x & \sigma_x^2\end{pmatrix}.$$

Finally, we define GDP as the sum of consumption, both investment types and the implicit rent from housing

$$\text{GDP}_t = Y_t + r_{H,t}H_t, \quad \text{where } r_{H,t} := \frac{\mu_H}{\mu_C}\frac{C_t}{H_t}.$$

We scale the variables in terms of technology A_{t-1} by $a_t := \frac{A_t}{A_{t-1}}$, $k_t^\star := \frac{K_t^\star}{A_{t-1}}$, $h_t^\star := \frac{H_t^\star}{A_{t-1}^{1-\varphi}}$, $k_t := \frac{K_t}{A_{t-1}}$, $h_t := \frac{H_t}{A_{t-1}^{1-\varphi}}$, $y_t := \frac{Y_t}{A_{t-1}}$, $c_t := \frac{C_t}{A_{t-1}}$, $\tilde{c}_t := \frac{\tilde{C}_t}{A_{t-1}}$, $i_t := \frac{I_t}{A_{t-1}}$, $d_t := \frac{D_t}{A_{t-1}}$, $w_t := \frac{W_t}{A_{t-1}}$, $p_{H,t} := \frac{P_{H,t}}{A_{t-1}^{\varphi}}$, $v_t := \frac{V_t}{A_{t-1}^{(1-1/\psi)}}$. Hence, the system of equations (18) and 19 can be written equivalently in terms of the scaled variables as

$$a_t = e^{a+x_t+\epsilon_{a,t}}, \tag{20a}$$

$$y_t = k_t^\alpha(a_t N_t)^{1-\alpha}, \tag{20b}$$

$$r_{K,t} = \alpha\frac{y_t}{k_t}, \tag{20c}$$

$$w_t = (1-\alpha)\frac{y_t}{N_t}, \tag{20d}$$

$$w_t = \frac{\mu_N}{\mu_C}\frac{c_t}{1-N_t}, \tag{20e}$$

$$y_t = c_t + i_t + d_t, \tag{20f}$$

$$\tilde{c}_t = c_t^{\mu_C} h_t^{\mu_H} (1 - N_t)^{\mu_N}, \tag{20g}$$

$$p_{H,t} = \frac{1}{1-\varphi} d_t^{\varphi}, \tag{20h}$$

$$q_t = \frac{1}{b_2} \left(\frac{i_t}{k_t} \right)^{\kappa}, \tag{20i}$$

$$a_t k_{t+1} = (1 - \delta_K) k_t + \Phi\left(\frac{i_t}{k_t} \right) k_t, \tag{20j}$$

$$a_t^{1-\varphi} h_{t+1} = (1 - \delta_H) h_t + d_t^{1-\varphi} \tag{20k}$$

$$q_t = \mathbb{E}_t \left[a_t^{-\frac{1}{\psi}} m_{t,t+1} \left(r_{K,t+1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta_K + \Phi\left(\frac{i_t}{k_t} \right) \right) \right) \right], \tag{20l}$$

$$p_{H,t} = \mathbb{E}_t \left[a_t^{-\frac{1}{\psi}+\varphi} m_{t,t+1} \left(\frac{\mu_H}{\mu_C} \frac{c_{t+1}}{h_{t+1}} + p_{H,t+1}(1 - \delta_H) \right) \right], \tag{20m}$$

$$v_t = (1-\beta) \tilde{c}_t^{1-\frac{1}{\psi}} + a_t^{1-\frac{1}{\psi}} \beta (\mathbb{E}_t v_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \tag{20n}$$

where

$$m_{t,t+1} = \beta \left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t} \right)^{1-\frac{1}{\psi}} \frac{c_t}{c_{t+1}} \left(\frac{v_{t+1}}{(\mathbb{E}_t v_{t+1}^{1-\theta})^{1/(1-\theta)}} \right)^{-\theta}.$$

Solution Method First, note that given period t's scaled state variables k_t, h_t, a_t, and x_t and the control variables for labor supply N_t, house prices $p_{H,t}$ and the value function v_t, all other period t variables as well as next period's endogenous state variables can be easily computed from equations (20a)-(20k). We approximate the policy functions for N_t, $p_{H,t}$ and the value function v_t by linear combinations of Chebyshev polynomials. We compute the coefficients in the linear combinations such way that the Euler equations (20l) and (20m) and the recursive equation (20n) for the value function are satisfied exactly at a sparse grid of collocation points (see Judd et al. (2014) and Heer and Maussner (2009) for details). Thereby, the expectations with respect to normally distributed random variables are computed by Gauss-Hermite quadrature.

B.3 Disaster risk

General Equilibrium Summing up, in any period t the economy's equilibrium is characterized by the following system of equations

$$K_t = e^{\omega_t b_t} K_t^{\star},$$

$$H_t = e^{(1-\varphi)\omega_t b_t} H_t^{\star},$$

$$Y_t = K_t^{\alpha} (A_t N_t)^{1-\alpha},$$

$$r_{K,t} = \alpha \frac{Y_t}{K_t},$$

$$
\begin{aligned}
W_t &= (1-\alpha)\frac{Y_t}{N_t}, \\
W_t &= \frac{\mu_N}{\mu_C}\frac{C_t}{1-N_t}, \\
Y_t &= C_t + I_t + D_t, \\
\tilde{C}_t &= C_t^{\mu_C}(A_{t-1}^{\varphi}H_t)^{\mu_H}(A_{t-1}(1-N_t))^{\mu_N}, \\
P_{H,t} &= \frac{1}{1-\varphi}D_t^{\varphi}, \\
q_t &= \frac{1}{b_2}\left(\frac{I_t}{K_t}\right)^{\kappa}, \\
K_{t+1}^{\star} &= (1-\delta_K)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t, \\
H_{t+1}^{\star} &= (1-\delta_H)H_t + D_t^{1-\varphi}, \\
q_t &= \mathbb{E}_t\left[M_{t,t+1}e^{\omega_{t+1}b_{t+1}}\left(r_{K,t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\left(1-\delta_K+\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right)\right], \\
P_{H,t} &= \mathbb{E}_t\left[M_{t,t+1}e^{(1-\varphi)\omega_{t+1}b_{t+1}}\left(\frac{\mu_H}{\mu_C}\frac{C_{t+1}}{H_{t+1}} + P_{H,t+1}(1-\delta_H)\right)\right], \\
V_t &= (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}},
\end{aligned}
$$

given the state variables $K_t^{\star}, H_t^{\star}, z_t, \omega_t, p_t$ and b_t. The SDF satisfies

$$
M_{t,t+1} = \beta\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{1-\frac{1}{\psi}}\frac{C_t}{C_{t+1}}\left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta}.
$$

Moreover, the growth rate satisfies

$$
\ln\left(\frac{A_{t+1}}{A_t}\right) = a + x_{t+1} + \omega_{t+1}b_{t+1},
$$

and the exogenous state variables are governed by the (independent) stochastic processes

$$
\begin{aligned}
& x_{t+1} = \rho_x x_t + \epsilon_{x,t+1},\ \epsilon_{x,t} \sim iid\mathcal{N}(0,\sigma_x^2), && \text{(22a)} \\
& \ln p_{t+1} = (1-\rho_p)\ln\bar{p} + \rho_p \ln p_t + \epsilon_{p,t+1},\ \epsilon_{p,t} \sim iid\mathcal{N}(0,\sigma_p^2), && \text{(22b)} \\
& \omega_t := \bar{\omega}e^{\hat{\omega}_t},\ \hat{\omega}_{t+1} = \rho_\omega\hat{\omega}_t + \epsilon_{\omega,t+1},\ \epsilon_{\omega,t} \sim iid\mathcal{N}\left(0,\sigma_\omega^2\right), && \text{(22c)} \\
& P(b_{t+1}=1|b_t=0) = \min\{p_t,1\},\ P(b_{t+1}=1|b_t=1) = \max\{q,\min\{p_t,1\}\}. && \text{(22d)}
\end{aligned}
$$

Finally, we define GDP as the sum of consumption, both investment types and the implicit rent from housing

$$
\text{GDP}_t = Y_t + r_{H,t}H_t, \quad \text{where } r_{H,t} := \frac{\mu_H}{\mu_C}\frac{C_t}{H_t}.
$$

We scale the variables in terms of technology A_{t-1} by $a_t := \frac{A_t}{A_{t-1}}$, $k_t^\star := \frac{K_t^\star}{A_{t-1}}$, $h_t^\star := \frac{H_t^\star}{A_{t-1}^{1-\varphi}}$, $k_t := \frac{K_t}{A_{t-1}}$, $h_t := \frac{H_t}{A_{t-1}^{1-\varphi}}$, $y_t := \frac{Y_t}{A_{t-1}}$, $c_t := \frac{C_t}{A_{t-1}}$, $\tilde{c}_t := \frac{\tilde{C}_t}{A_{t-1}}$ $i_t := \frac{I_t}{A_{t-1}}$, $d_t := \frac{D_t}{A_{t-1}}$, $w_t := \frac{W_t}{A_{t-1}}$, $p_{H,t} := \frac{P_{H,t}}{A_{t-1}^{\varphi}}$, $v_t := \frac{V_t}{A_{t-1}^{(1-1/\psi)}}$. Hence, the system of equations (21) can be written equivalently in terms of the scaled variables as

$$a_t = e^{a+x_t+\omega_t b_t}, \tag{23a}$$

$$k_t = e^{\omega_t b_t} k_t^\star, \tag{23b}$$

$$h_t = e^{(1-\varphi)\omega_t b_t} h_t^\star, \tag{23c}$$

$$y_t = k_t^\alpha (a_t N_t)^{1-\alpha}, \tag{23d}$$

$$r_{K,t} = \alpha \frac{y_t}{k_t}, \tag{23e}$$

$$w_t = (1-\alpha)\frac{y_t}{N_t}, \tag{23f}$$

$$w_t = \frac{\mu_N}{\mu_C}\frac{c_t}{1-N_t}, \tag{23g}$$

$$y_t = c_t + i_t + d_t, \tag{23h}$$

$$\tilde{c}_t = c_t^{\mu_C} h_t^{\mu_H} (1-N_t)^{\mu_N}, \tag{23i}$$

$$p_{H,t} = \frac{1}{1-\varphi} d_t^{\varphi}, \tag{23j}$$

$$q_t = \frac{1}{b_2}\left(\frac{i_t}{k_t}\right)^{\kappa}, \tag{23k}$$

$$a_t k_{t+1}^\star = (1-\delta_K)k_t + \Phi\left(\frac{i_t}{k_t}\right)k_t, \tag{23l}$$

$$a_t^{1-\varphi} h_{t+1}^\star = (1-\delta_H)h_t + d_t^{1-\varphi} \tag{23m}$$

$$q_t = \mathbb{E}_t\left[a_t^{-\frac{1}{\psi}} m_{t,t+1} e^{\omega_{t+1}b_{t+1}}\left(r_{K,t+1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1}\left(1-\delta_K + \Phi\left(\frac{i_t}{k_t}\right)\right)\right)\right], \tag{23n}$$

$$p_{H,t} = \mathbb{E}_t\left[a_t^{-\frac{1}{\psi}+\varphi} m_{t,t+1} e^{(1-\varphi)\omega_{t+1}b_{t+1}}\left(\frac{\mu_H}{\mu_C}\frac{c_{t+1}}{h_{t+1}} + p_{H,t+1}(1-\delta_H)\right)\right], \tag{23o}$$

$$v_t = (1-\beta)\tilde{c}_t^{1-\frac{1}{\psi}} + a_t^{(1-1/\psi)}\beta(\mathbb{E}_t v_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \tag{23p}$$

where

$$m_{t,t+1} = \beta\left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{1-\frac{1}{\psi}} \frac{c_t}{c_{t+1}}\left(\frac{v_{t+1}}{(\mathbb{E}_t v_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta}.$$

Solution Method First, note that given period t's scaled state variables $k_t^\star, h_t^\star, x_t, \omega_t, p_t$ and b_t and the control variables for labor supply N_t, house prices $p_{H,t}$ and the value function v_t, all other period t variables as well as next period's endogenous state variables can be easily computed from equations (23a)-(23m). We approximate the policy functions for N_t, $p_{H,t}$ and the value function v_t by linear combinations of Chebyshev polynomials. We compute the coefficients in the linear combinations such way that the Euler equations (23n) and (23o) and the recursive equation (23p) for the value function are satisfied exactly at a sparse grid of collocation points (see Judd et al. (2014) and Heer and Maussner (2009) for details). Thereby, the expectations with respect to normally distributed random variables are computed by Gauss-Hermite quadrature.